

antropy: native multidimensional entropy and configurable complexity analysis

Methods, replicated synthetic studies, and resource-aware reproducibility

Daniel Ari Friedman
Active Inference Institute
`daniel@activeinference.institute`

Contents

Abstract	2
1 Introduction	2
2 Scope and related work	3
3 Methodology	3
3.1 Complete registered interface	3
3.2 Native spatial templates	4
3.3 High-dimensional sample entropy	4
3.4 Related local-template estimators	5
3.5 Probability, ordinal and decomposition estimators	5
3.6 Jensen-Shannon statistical complexity	6
4 Experimental setup	7
4.1 Retained baseline	7
4.2 Replicated Bernoulli-replacement design	7
4.3 Geometry and repetition	10
5 Computational complexity and resources	10
5.1 Empirical timing observations	11
6 Computational results	13
6.1 Baseline outcomes	13
6.2 Replicated native HDSE	13
6.3 Structured controls and alternative representations	14
6.4 Related entropy and statistical complexity	19
7 Inference and prospective operating characteristics	22
7.1 Scientific unit and declared design	22
7.2 Permutation decisions and multiplicity	24
7.3 Availability, rejection rates and uncertainty	25
7.4 Effect summaries and bootstrap interface	40
8 Reproducibility and software architecture	44
8.1 Numerical and file boundaries	44
8.2 Source-bound manuscript generation	44
8.3 Provenance bindings	45
9 Discussion and conclusions	45
References	46

Abstract

antropy: native multidimensional entropy and configurable complexity analysis provides a configurable framework for native spatial entropy, probability information, ordinal patterns, spectral distributions and trajectory structure. The implementation separates local spatial extraction from pair selection and estimator semantics. A common typed result preserves finite, undefined and infinite outcomes instead of collapsing them into missing numeric values. This manuscript accompanies antropy-hd 0.3.1.

The retained baseline contains 54 measured rows. The configured replicated study contains 150 fields and 1650 scalar estimates, including native HDSE, row-major flattened sample entropy and a fixed-alphabet entropy-complexity plane. Its field sizes are 48, 10 x 12, 6 x 7 x 8, with 5 independent generated trials per condition. Analytical work estimates report actual blocked-kernel schedules; empirical runtimes retain their host and repetition metadata. Source variables, figures and tables are generated from the stored reports and validated configuration.

A separate prospective simulation retains 18432 whole-field estimator records, 6912 inference decisions and 108 operating-characteristic cells. The configured procedure compares 4 estimators under holm multiplicity control at significance level 0.05, with pointwise 0.95 confidence intervals. Availability and selected rejection retain separate denominators; these synthetic operating characteristics do not certify a scientific sample size.

The published HDSE construction motivates the local-template estimator, while the manuscript distinguishes source conventions, ambiguities and explicit extensions. The experiments are controlled synthetic demonstrations rather than a reproduction of the source paper’s empirical classification results. Rendering uses the pinned shared research template’s Pandoc, LaTeX, bibliography and real table-of-contents pipeline.

1 Introduction

Entropy is a family of questions about a specified representation, population and uncertainty model. A probability entropy asks about masses over an alphabet. A sample-template estimator asks how frequently similar local configurations remain similar after their representation is extended. Ordinal and spectral estimators construct different distributions. Their outputs require the input meaning, units, normalization and finite-sample convention to remain visible.

Native multidimensional data make these distinctions especially consequential. A spatial field has coordinate adjacency along several axes. Globally flattening it changes which values become adjacent in a one-dimensional embedding. Native HDSE extracts rectangular local blocks before their values are arranged for distance computation [Xia and Wang, 2026]. The mathematical estimator and the file representation are therefore distinct layers.

The repository name combines entropy and ant, reflecting interest in collective biological organization. No biological observations are included in the present study. The distribution and Python import use the separate names `antropy-hd` and `antropy_hd` to preserve compatibility with the unrelated AntroPy time-series library. The current registry exposes 19 orchestratable entries, while the numerical API also exposes reusable geometry, information and cost-estimation components.

The project has four connected aims: state estimator contracts precisely; preserve native geometry and explicit degeneracy; produce a replicated synthetic study with complete result coverage; and bind measured outputs to executable, resource-aware software and a configurable manuscript. The source repository is [the configured public project](#).

2 Scope and related work

Shannon’s discrete entropy defines information over a probability alphabet [Shannon, 1948]. Renyi and Tsallis generalize that functional while changing its order or extensivity conventions [Renyi, 1961, Tsallis, 1988]. Joint entropy, conditional entropy and mutual information retain the variable-axis meanings of probability tensors. Jensen-Shannon divergence compares distributions over a shared alphabet [Lin, 1991]. These identities do not choose an amplitude discretization or establish a spatial embedding.

Approximate entropy and sample entropy make local-pattern continuation central [Pincus, 1991, Richman and Moorman, 2000]. HDSE extends local sample-template reasoning to higher-dimensional fields [Xia and Wang, 2026]. Fuzzy entropy replaces a discontinuous threshold indicator with a specified similarity kernel [Chen et al., 2007]. Multiscale entropy asks related questions after coarse graining [Costa et al., 2002]. Distribution entropy summarizes the empirical distribution of template distances [Li et al., 2015].

Permutation entropy constructs ordinal-pattern probabilities, with weighted permutation entropy retaining local amplitude variability [Bandt and Pompe, 2002, Fadlallah et al., 2013]. Spectral and trajectory decompositions provide additional representations [Rezek and Roberts, 1998, Broomhead and King, 1986]. The implementation uses a full multidimensional FFT periodogram and normalized singular values; neither convention is assumed identical to every spectral or SVD estimator in use.

Statistical complexity combines normalized entropy with disequilibrium. Here the Jensen-Shannon construction follows Rosso and colleagues and the broader family described by Martin and colleagues [Rosso et al., 2007, Martin et al., 2006]. Fixed-bin amplitude probabilities are a declared descriptive extension. This study does not infer Bandt-Pompe causality or diagnose noise versus chaos from those amplitudes.

The unified framework is an engineering and mathematical organization of these contracts. It does not assert that the estimators define one calibrated scalar ordering of complexity. Native HDSE versus flattened SampEn also changes feature count and extension geometry; that comparison is not a controlled intervention on flattening alone.

3 Methodology

3.1 Complete registered interface

The following table is generated from the current executable registry and call signatures. Arguments select mathematical conventions and resource limits; all numerical claims use the effective configuration recorded with each result. Input probabilities and raw field observations have separate meanings.

Registered estimator	Input meaning	Configured arguments
approximate entropy	observations	embedding, match
conditional entropy	joint probabilities	conditioned axes, base
distribution entropy	observations	embedding, match, bins, normalize
fuzzy entropy	observations	embedding, match, power, center
hdse	observations	embedding, match
histogram entropy	observations	bins, base, max bins
jensen shannon divergence	probabilities; q supplied as parameter	q, weight, base

Registered estimator	Input meaning	Configured arguments
joint entropy	joint probabilities	base
multiscale entropy	observations	scales, embedding, match, preserve tolerance
mutual information	joint probabilities	x axes, base
permutation entropy	observations	embedding, weighted, base, normalize, max pattern length
renyi entropy	probabilities	order, base, normalize
sample entropy	observations	embedding, match
shannon entropy	probabilities	base, normalize
spectral entropy	observations	demean, include dc, base, normalize, max memory bytes
statistical complexity	fixed-alphabet probabilities	config
svd entropy	observations	embedding, demean, base, normalize
tsallis entropy	probabilities	order, normalize
weighted permutation entropy	observations	embedding, base, normalize, max pattern length

3.2 Native spatial templates

Let x be a finite real-valued array with axis lengths $L = (L_1, \dots, L_d)$. An embedding specifies block shape m , delays δ , strides s and extension axes. For an origin i , the local template contains $x_{i+k\odot\delta}$ for $0 \leq k_j < m_j$. Only this extracted block is flattened, in C order. Its feature count is $q = \prod_j m_j$.

Write m^+ for the extended shape. Common origins are chosen from the geometry supporting that extended block, so the paired population has

$$n = \prod_{j=1}^d \max \left(0, 1 + \left\lfloor \frac{L_j - 1 - (m_j^+ - 1)\delta_j}{s_j} \right\rfloor \right). \quad (1)$$

The numerical boundary rejects nonfinite, complex, boolean, string, object, empty and scalar inputs. Geometry and resource failures raise typed errors. Rectangular shapes, selected extension axes, delays and strides are explicit extensions of the paper’s common isotropic window construction.

3.3 High-dimensional sample entropy

For paired base and extended templates, use the same admissible unordered origin pair set $\mathcal{P}$. With Chebyshev distance $D_\infty(u, v) = \max_k |u_k - v_k|$ and threshold $\tau = r\sigma_x$, define

$$B = \sum_{(i,j) \in \mathcal{P}} \mathbf{1}\{D_\infty(u_i, u_j) < \tau\}, \quad A = \sum_{(i,j) \in \mathcal{P}} \mathbf{1}\{D_\infty(u_i^+, u_j^+) < \tau\}. \quad (2)$$

When $B > 0$ and $A > 0$,

$$\text{HDSE}(x) = -\ln(A/B). \quad (3)$$

The implementation counts unordered pairs; doubling numerator and denominator recovers the corresponding ordered symmetric counting convention. The standard deviation is computed over

the complete input, with population variance by default because the source does not specify the degrees-of-freedom convention. Strict thresholding follows the published HDSE definition [Xia and Wang, 2026].

The paper’s prose requiring every origin coordinate to differ and its equations excluding only the identical origin tuple produce distinct populations. Ordinary tuple self-exclusion is the default. Coordinate exclusion is exposed as a documented sensitivity choice, along with Theiler exclusions and other distance norms. For HDSE, $B = 0$ is undefined; $B > 0, A = 0$ is infinite. A constant field has zero relative threshold, so strict matching yields no matches even when its amplitudes are identical.

The executable procedure is:

1. Parse the real array and immutable configuration.
2. Resolve common valid origins and guard template/pair storage.
3. Extract native base and extended windows on those origins.
4. Resolve global tolerance once from the declared standard deviation.
5. Count both representations with the same pair-selection policy.
6. Return value, status, populations, matching counts and effective parameters.

3.4 Related local-template estimators

One-dimensional SampEn uses the same paired engine with its separately declared boundary convention [Richman and Moorman, 2000]. Approximate entropy uses complete, separate origin populations at each block size and includes self matches. Its statistic is $\phi_m - \phi_{m+1}$, where $\phi_m = n_m^{-1} \sum_i \ln C_i^{(m)}$ [Pincus, 1991]. These populations and normalizers are not substituted into HDSE.

Fuzzy entropy uses the explicit kernel $K(D) = \exp[-(D/\tau)^p]$, optionally centering each template, and accumulates membership sums in the log domain [Chen et al., 2007]. Centering changes template semantics, so fuzzy continuation need not share all monotonicity properties of un-centered threshold counts. Nonfinite eligible distances or log weights raise a numerical-range error rather than becoming mathematical undefined estimates. Its statistic is the difference between the logarithms of mean base and extended pair memberships, with the same declared pair-selection policy.

Multiscale analysis uses trimmed block means at the configured scale vector [Costa et al., 2002]. Original-input tolerance is preserved by default. Distribution entropy uses a specified histogram of pairwise distances; using all valid local origins, including the final valid origin omitted by the original DistEn construction, is explicitly an implementation extension [Li et al., 2015].

For scale vector c , coarse-grained observations are

$$y_\ell = \frac{1}{\prod_j c_j} \sum_{0 \leq u_j < c_j} x_{\ell \odot c + u}. \quad (4)$$

Partial outer blocks are trimmed. The implementation evaluates HDSE at each coarse-grained level, extending the original one-dimensional multiscale construction to native spatial fields. Distribution entropy instead forms probabilities from the histogram of eligible pair distances: $H_D = -\sum_{k=1}^K p_k \ln p_k$, optionally divided by $\ln K$ for $K > 1$.

3.5 Probability, ordinal and decomposition estimators

For a declared probability alphabet of size M ,

$$H_b(P) = - \sum_{a=1}^M p_a \log_b p_a, \quad (5)$$

with zero-mass terms contributing zero. Renyi entropy is $\log_b(\sum_a p_a^\alpha)/(1 - \alpha)$, with explicit support-size, Shannon and min-entropy limits. Tsallis entropy is $(1 - \sum_a p_a^\alpha)/(\alpha - 1)$ with $k = 1$ and a natural-log Shannon limit [Renyi, 1961, Tsallis, 1988]. Conditional entropy and mutual information use the named tensor-axis partitions, not the spatial-axis interpretation of an amplitude field.

For a joint probability array $P_{X,Y}$, marginalization sums over the declared complementary axes. The joint entropy is the Shannon entropy of the complete joint alphabet. The implemented identities are

$$H_b(X | Y) = H_b(X, Y) - H_b(Y), \quad I_b(X; Y) = H_b(X) + H_b(Y) - H_b(X, Y). \quad (6)$$

Amplitude histogram entropy first constructs probabilities from the configured bins and every observed amplitude, then applies H_b . Bin edges that exclude observed amplitudes are rejected rather than silently truncating the field. For two distributions on the same alphabet, the configurable weighted Jensen-Shannon divergence is

$$J_{b,w}(P, Q) = H_b(wP + (1 - w)Q) - wH_b(P) - (1 - w)H_b(Q). \quad (7)$$

Permutation entropy applies Shannon entropy to local ordinal-pattern probabilities; equal amplitudes are ordered stably by local C-order position [Bandt and Pompe, 2002]. Its weighted version uses local population variance [Fadlallah et al., 2013]. Spectral entropy normalizes the complete multidimensional FFT power distribution, retaining conjugate bins individually. SVD entropy normalizes singular values of the local trajectory matrix without squaring them [Rezek and Roberts, 1998, Broomhead and King, 1986]. Units, normalization and alphabet sizes accompany each result; singleton and zero-energy degeneracies are handled explicitly.

If local windows have patterns π_i and population variances v_i , weighted permutation probabilities are $p_\pi = \sum_i v_i \mathbf{1}\{\pi_i = \pi\} / \sum_i v_i$. Ordinary permutation probabilities replace these variance weights with unit weights. Their normalized entropy uses the declared factorial pattern alphabet, not only the observed patterns. Spectral probabilities are $p_k = |\mathcal{F}(x)_k|^2 / \sum_j |\mathcal{F}(x)_j|^2$, with the configured mean/DC policy. Trajectory SVD probabilities are $p_i = \sigma_i / \sum_j \sigma_j$ for singular values σ_i of the configured local-window matrix. These constructions apply Shannon entropy to different representations; equal numerical values do not imply equal information about spatial structure.

3.6 Jensen-Shannon statistical complexity

Let P_e be uniform on the complete configured alphabet and Δ_M a point mass on that same alphabet. The statistic is

$$C_{JS}(P) = \frac{H_b(P)}{\log_b M} \frac{J_b(P, P_e)}{J_b(\Delta_M, P_e)}, \quad J_b(P, Q) = H_b\left(\frac{P + Q}{2}\right) - \frac{H_b(P) + H_b(Q)}{2}. \quad (8)$$

Zero-probability cells remain in the declared alphabet. A point mass has zero normalized entropy; a uniform distribution has zero disequilibrium. Their statistical complexities are therefore zero for different reasons [Rosso et al., 2007, Martin et al., 2006]. The final statistic is dimensionless. It complements entropy rather than acting as its renamed value.

The study’s fixed-bin amplitude alphabet has $M - 2$ interior intervals and two overflow cells, giving M total categories. Values below the lower bound or at least the upper bound remain in the corresponding overflow cell; empty cells remain in the probability vector. This symbolization is a declared descriptive extension, not a causal ordinal partition.

4 Experimental setup

4.1 Retained baseline

The original implementation demonstration is retained as a distinct dataset. Its shapes are 64, 16 x 16, 8 x 8 x 8, embedding shape 2, relative radii 0.2, 0.5, 1 and seed 20261009. It uses periodic, Gaussian and additive-mixture fields with period 8 and mixture amplitude 0.35. This smaller demonstration is not pooled with the replicated study.

4.2 Replicated Bernoulli-replacement design

The main study uses shapes 48, 10 x 12, 6 x 7 x 8, embedding shape 2, relative radii 1.5, 2, 2.5 and mixing probabilities 0, 0.25, 0.5, 0.75, 1. Each condition has 5 generated trials. The period is 12 and the root seed is 20261009. The complete consumed design is generated below from the actual stored plan.

Control	Configured value
schema version	1
seed	20261009
shapes	[[48], [10, 12], [6, 7, 8]]
mixing probabilities	[0.0, 0.25, 0.5, 0.75, 1.0]
controls	["constant", "checkerboard", "gaussian", "correlated", "shuffled"]
replicates	5
radii	[1.5, 2.0, 2.5]
period	12.0
phase origin	1
embedding shape	2
gaussian sd	1.0
correlation steps	3
symbol range	[-4.0, 4.0]
symbol bins	16
max input elements	10000
max fields	200
max distance component visits	500000000
max templates	1000
max pairs	500000
max memory bytes	134217728
block size	64

The MIX construction follows the source paper’s replacement form rather than the baseline’s additive Gaussian mixture [Xia and Wang, 2026]. For coordinate u ,

$$s(u) = \sum_{j=1}^d \sin(2\pi(u_j + u_0)/T), \quad x_p(u) = (1 - \xi_{p,u})s(u) + \xi_{p,u}\eta_u, \quad (9)$$

where $u_0 = 1$ is the configured coordinate phase origin, ξ is Bernoulli with parameter p and η is uniform on $[-\sqrt{3}, \sqrt{3}]$. The periodic sum is not divided by $\sqrt{d}$. Common random numbers couple the mixing sweep within a trial, while the trial seed schedule separates generated fields. The design is bounded by explicit template, pair, input-element and workspace budgets.

Controls are constant, checkerboard, gaussian, correlated, shuffled. The Gaussian scale is 1; the configured correlation procedure uses 3 steps. Fixed amplitude symbolization declares 16 cells over -4.0, 4.0. Complete probability vectors include empty cells. Input meaning and symbolization policy are therefore part of the recorded configuration rather than inferred from a plot.

Actual trial-0 inputs; MIX $p=0.5$; dimensions >2 use central slices

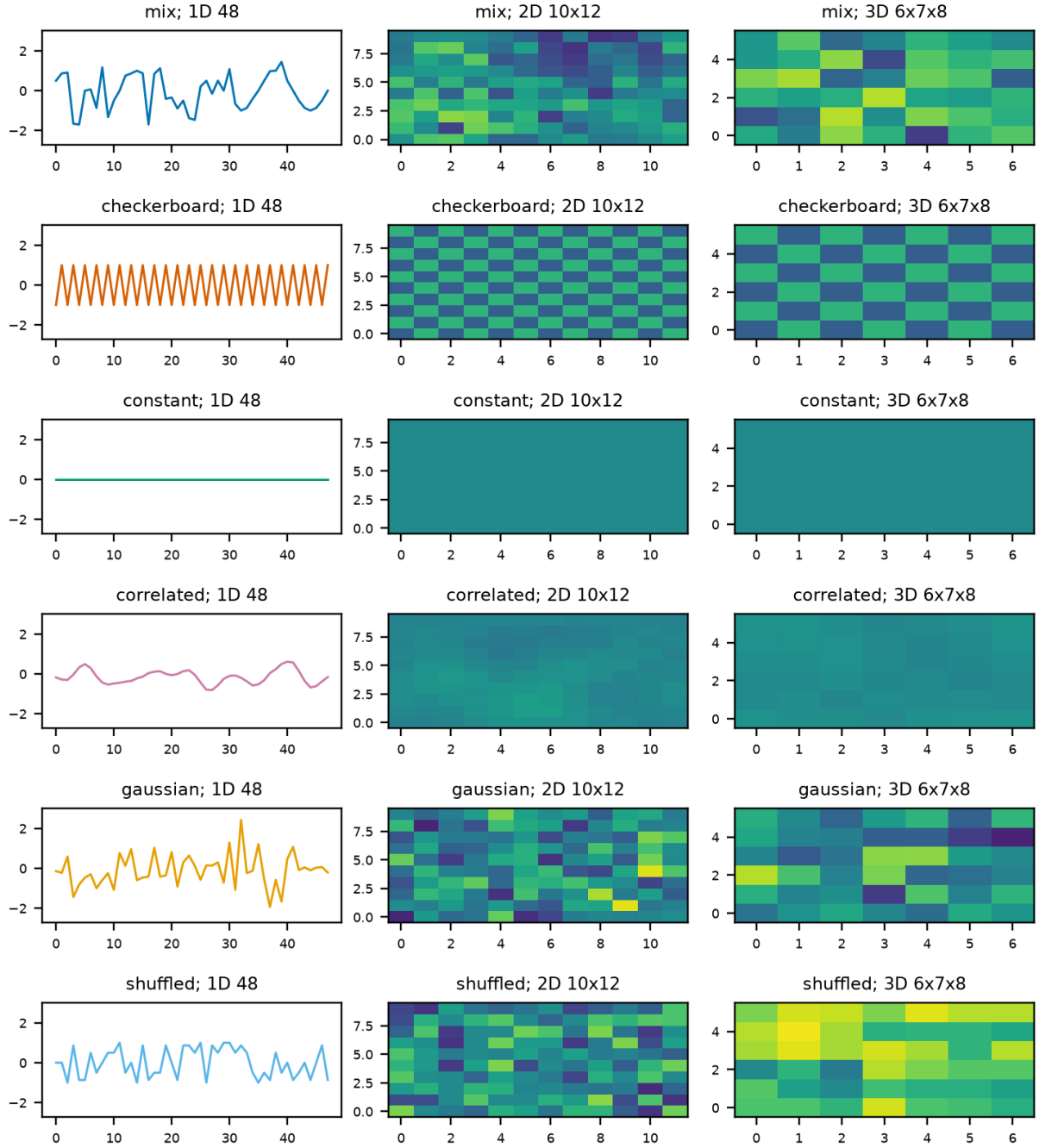


Figure 1: Illustrative generated fields from the stored synthetic design. Full inputs and seeds are preserved separately from their visual examples.

4.3 Geometry and repetition

Native HDSE and row-major flattened SampEn receive the same underlying amplitude field and relative-radius setting, but their local representations differ. A native block has m^d base features and an all-axis extension has $(m+1)^d$; the flattened one-dimensional continuation adds one feature. Origin populations and continuation dimensions also differ. These are comparison conditions, not an assumption that the estimators must agree.

Reported means and sample standard deviations condition on finite outcomes. Every summary separately retains total, finite, undefined and infinite counts. Generated trials are the unit of variation; overlapping template pairs are not treated as independent experimental replicates. No significance test or classification accuracy is inferred from this synthetic design. Deterministic conditions can produce identical fields across trial identities; their repeated values do not establish empirical uncertainty.

5 Computational complexity and resources

For n local templates with q components, explicit extraction requires $O(nq)$ component work and storage. Exhaustive pair comparison contributes $O(n^2q)$ distance-component work. Extending all axes replaces q by $q^+ = \prod_j (m_j + 1)$, so native-dimensionality changes feature count as well as the valid-origin population in eq. 1. Flattening changes both quantities.

The implementation processes bounded pair blocks. This controls workspace but does not remove quadratic total pair work. Its allocation-free estimator reports the actual scheduled distance cells, component visits, blocks, origin arrays and conservative guard bytes for the selected request. Distance work can include cells later excluded by pair masks; admissible pair counts and computed distance cells are distinct quantities. Exact scheduled counts, asymptotic size terms and host runtime observations retain separate interpretations.

Identical scheduled-work rows across radius settings are listed once below; the complete per-analysis estimates remain in the stored complexity report.

Here flat denotes row-major flattened observations; maximum and histogram denote the distance passes.

Input	Stage	n	q	Pairs	Visits	Guard (B)
6 x 7 x 8	base native	120	8	7140	86528	695936
6 x 7 x 8	extended native	120	27	7140	292032	695936
6 x 7 x 8	maximum native	210	8	21945	226848	683056
6 x 7 x 8	histogram native	210	8	21945	226848	683056
6 x 7 x 8	local native	210	8	0	0	85680
336	base flat	334	2	55611	132232	675488
336	extended flat	334	3	55611	198348	675488
10 x 12	base native	80	4	3160	21504	669056
10 x 12	extended native	80	9	3160	48384	669056
10 x 12	maximum native	99	4	4851	30244	669328
10 x 12	histogram native	99	4	4851	30244	669328
10 x 12	local native	99	4	0	0	20592
120	base flat	118	2	6903	20936	665120
120	extended flat	118	3	6903	31404	665120
48	base native	46	2	1035	4232	344864
48	extended native	46	3	1035	6348	344864
48	maximum native	47	2	1081	4418	363784
48	histogram native	47	2	1081	4418	363784
48	local native	47	2	0	0	7120
48	base flat	46	2	1035	4232	344864

Input	Stage	n	q	Pairs	Visits	Guard (B)
48	extended flat	46	3	1035	6348	344864

Guard bytes cover the controlled template and pair workspace. Caller-owned input arrays, file loading, standard-deviation temporaries, allocator overhead, vendor-library workspaces and process resident memory are outside that bound. The guard must reject infeasible requests; it is not a measurement of process RSS or a universal memory guarantee.

5.1 Empirical timing observations

The recorded environment is Python 3.14.7, NumPy 2.5.3, macOS-27.0.1-arm64-arm-64bit-Mach-O, machine arm64. The configured plan uses 1 warmups and 3 measured repetitions per request; block sizes are 16, 64.

Shape	Block	Median (s)	Sample SD (s)	Visits	Guard (B)
4 x 4 x 4	16	0.000233416	3.24125e-05	2240	16768
4 x 4 x 4	64	0.000179125	6.43296e-06	2240	16768
6 x 6	16	0.000105208	2.92041e-06	3328	46976
6 x 6	64	0.000101584	4.27311e-06	3328	46976
6 x 6 x 6	16	0.000730875	4.06458e-06	89600	64512
6 x 6 x 6	64	0.000358125	3.68909e-06	143360	678912
8 x 8 x 8	16	0.00741188	0.000489106	875840	110720
8 x 8 x 8	64	0.00221696	0.000139051	1041600	725120
10 x 10	16	0.000394875	2.604e-05	33280	52736
10 x 10	64	0.000279375	5.43607e-05	53248	667136
14 x 14	16	0.00151417	8.36652e-05	149760	62336
14 x 14	64	0.000536333	0.000197742	189696	676736
32	16	0.000118458	8.8355e-06	3380	46496
32	64	9.5167e-05	5.26603e-05	4500	149536
64	16	0.00025375	5.37063e-05	12020	48032
64	64	0.000103292	3.42341e-06	19220	622112
96	16	0.000437917	8.76008e-06	25780	49568
96	64	0.000261333	7.39915e-05	34580	663968

Timings are observed on the recorded environment and inputs. They do not prove an asymptotic theorem or promise portable latency. Block size can change workspace, call scheduling and run-time without changing the mathematical pair population or estimator value. Numerical validity, analytical cost and measured runtime are therefore checked and reported independently.

Real HDSE calls on arm64; bars: repetition sample SD

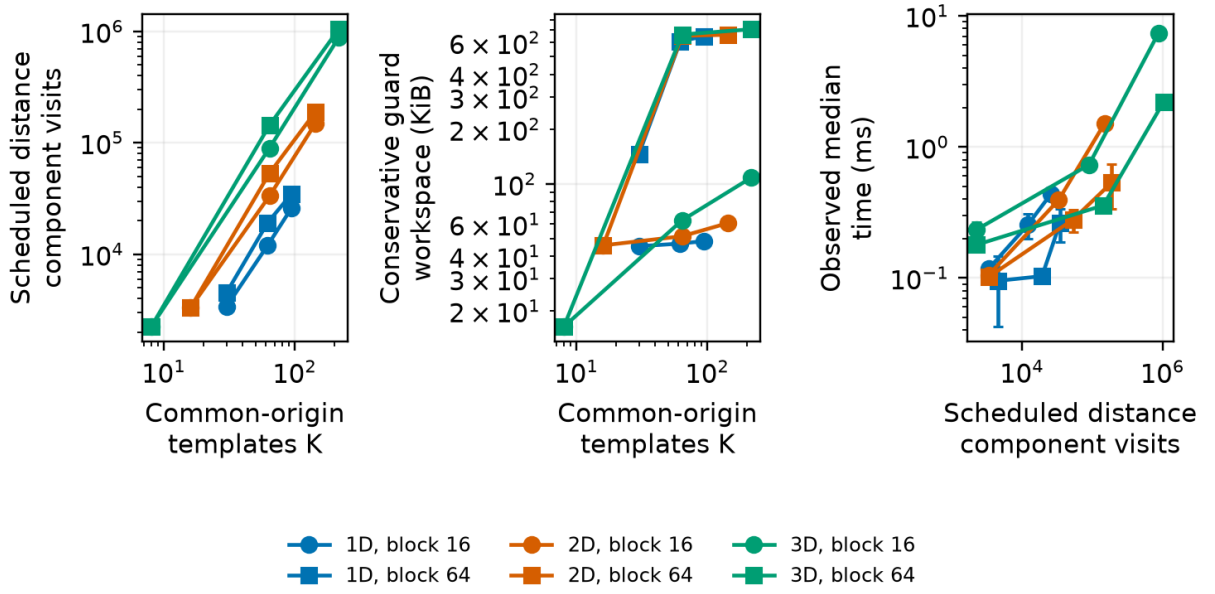


Figure 2: Measured runtime and configured deterministic work estimates. Host timings are diagnostic observations; exact scheduled counts and asymptotic terms have separate interpretations.

6 Computational results

6.1 Baseline outcomes

The baseline records 54 estimates: 45 finite, 3 undefined and 6 infinite. Its HDSE values are displayed directly from the stored CSV.

Shape	Field	r	HDSE (nats)	Status
64	periodic	0.2	0	finite
64	periodic	0.5	0.451985	finite
64	periodic	1	0.245482	finite
64	noise	0.2	1.4816	finite
64	noise	0.5	1.22536	finite
64	noise	1	0.680827	finite
64	mixed	0.2	2.70805	finite
64	mixed	0.5	1.02233	finite
64	mixed	1	0.488638	finite
16x16	periodic	0.2	0.262364	finite
16x16	periodic	0.5	1.07654	finite
16x16	periodic	1	0.765302	finite
16x16	noise	0.2	-	undefined
16x16	noise	0.5	-	infinite
16x16	noise	1	3.31381	finite
16x16	mixed	0.2	-	infinite
16x16	mixed	0.5	3.17109	finite
16x16	mixed	1	1.75318	finite
8x8x8	periodic	0.2	-	infinite
8x8x8	periodic	0.5	1.86322	finite
8x8x8	periodic	1	0.898039	finite
8x8x8	noise	0.2	-	undefined
8x8x8	noise	0.5	-	infinite
8x8x8	noise	1	-	infinite
8x8x8	mixed	0.2	-	undefined
8x8x8	mixed	0.5	-	infinite
8x8x8	mixed	1	4.71253	finite

6.2 Replicated native HDSE

The replicated report contains 150 fields and 1650 estimates: 1528 finite, 105 undefined and 17 infinite. The tables retain the finite conditional mean and sample SD, the finite fraction and the undefined/infinite counts (U/I). A dash indicates an unavailable finite mean or SD; it does not denote zero entropy.

Shape	p	r	Mean	SD	Finite/n	U/I
6x7x8	0	1.5	0.620344	0	5/5	0/0
6x7x8	0	2	0.334022	0	5/5	0/0
6x7x8	0	2.5	0.181913	0	5/5	0/0
6x7x8	0.25	1.5	3.71297	1.44226	5/5	0/0
6x7x8	0.25	2	2.21524	0.731486	5/5	0/0
6x7x8	0.25	2.5	1.18716	0.254853	5/5	0/0

Shape	p	r	Mean	SD	Finite/n	U/I
6x7x8	0.5	1.5	5.22817	0.343375	3/5	0/2
6x7x8	0.5	2	2.40793	0.402156	5/5	0/0
6x7x8	0.5	2.5	1.14763	0.136496	5/5	0/0
6x7x8	0.75	1.5	5.2898	0.184894	2/5	0/3
6x7x8	0.75	2	3.29457	0.316691	5/5	0/0
6x7x8	0.75	2.5	1.44975	0.191395	5/5	0/0
6x7x8	1	1.5	-	-	0/5	0/5
6x7x8	1	2	4.14886	0.305753	5/5	0/0
6x7x8	1	2.5	1.68958	0.112868	5/5	0/0
10x12	0	1.5	0.195222	0	5/5	0/0
10x12	0	2	0.0950618	0	5/5	0/0
10x12	0	2.5	0.0471768	0	5/5	0/0
10x12	0.25	1.5	0.845984	0.198192	5/5	0/0
10x12	0.25	2	0.412154	0.0775165	5/5	0/0
10x12	0.25	2.5	0.204453	0.0403457	5/5	0/0
10x12	0.5	1.5	1.39016	0.187022	5/5	0/0
10x12	0.5	2	0.637592	0.0698127	5/5	0/0
10x12	0.5	2.5	0.301306	0.0347635	5/5	0/0
10x12	0.75	1.5	1.84024	0.161139	5/5	0/0
10x12	0.75	2	0.977935	0.0408192	5/5	0/0
10x12	0.75	2.5	0.434886	0.0426958	5/5	0/0
10x12	1	1.5	2.00003	0.179567	5/5	0/0
10x12	1	2	1.06436	0.127857	5/5	0/0
10x12	1	2.5	0.425303	0.0656491	5/5	0/0
48	0	1.5	0.163193	0	5/5	0/0
48	0	2	0.0943835	0	5/5	0/0
48	0	2.5	0.0497278	0	5/5	0/0
48	0.25	1.5	0.34872	0.0451071	5/5	0/0
48	0.25	2	0.190946	0.0265122	5/5	0/0
48	0.25	2.5	0.0761487	0.0129646	5/5	0/0
48	0.5	1.5	0.395644	0.0426951	5/5	0/0
48	0.5	2	0.189375	0.0111342	5/5	0/0
48	0.5	2.5	0.0752926	0.00613628	5/5	0/0
48	0.75	1.5	0.387622	0.0279628	5/5	0/0
48	0.75	2	0.196965	0.023594	5/5	0/0
48	0.75	2.5	0.0802966	0.00841765	5/5	0/0
48	1	1.5	0.415829	0.022104	5/5	0/0
48	1	2	0.237607	0.0212663	5/5	0/0
48	1	2.5	0.0810962	0.015752	5/5	0/0

Matching probabilities provide another view of these outcomes. A positive base count followed by zero extended matches gives infinite continuation entropy; zero base matches give undefined entropy. Increasing native feature count and changing spatial geometry can therefore affect interpretability even when all input amplitudes are valid.

6.3 Structured controls and alternative representations

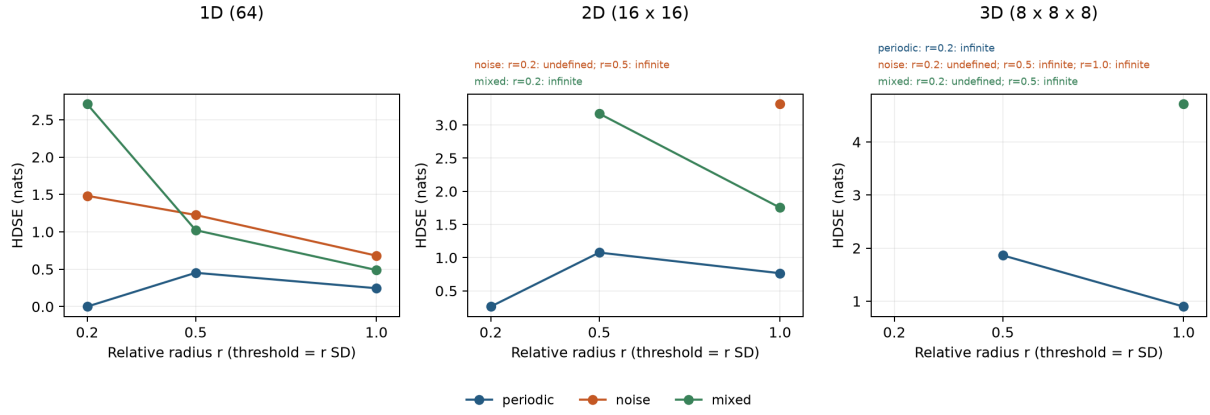


Figure 3: Baseline HDSE tolerance demonstration. Finite estimates are plotted; undefined and infinite estimates remain annotated and recorded in the table.

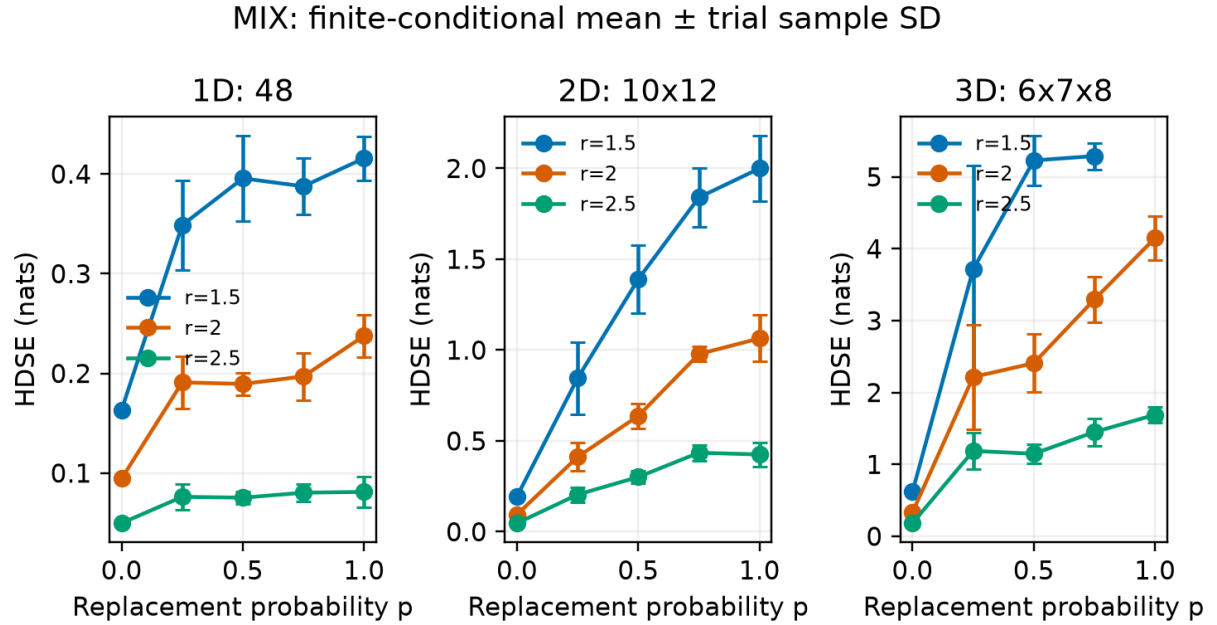


Figure 4: Native HDSE across Bernoulli replacement probabilities and configured radii. Finite conditional summaries are accompanied by outcome counts; variability is across generated fields, not template pairs.

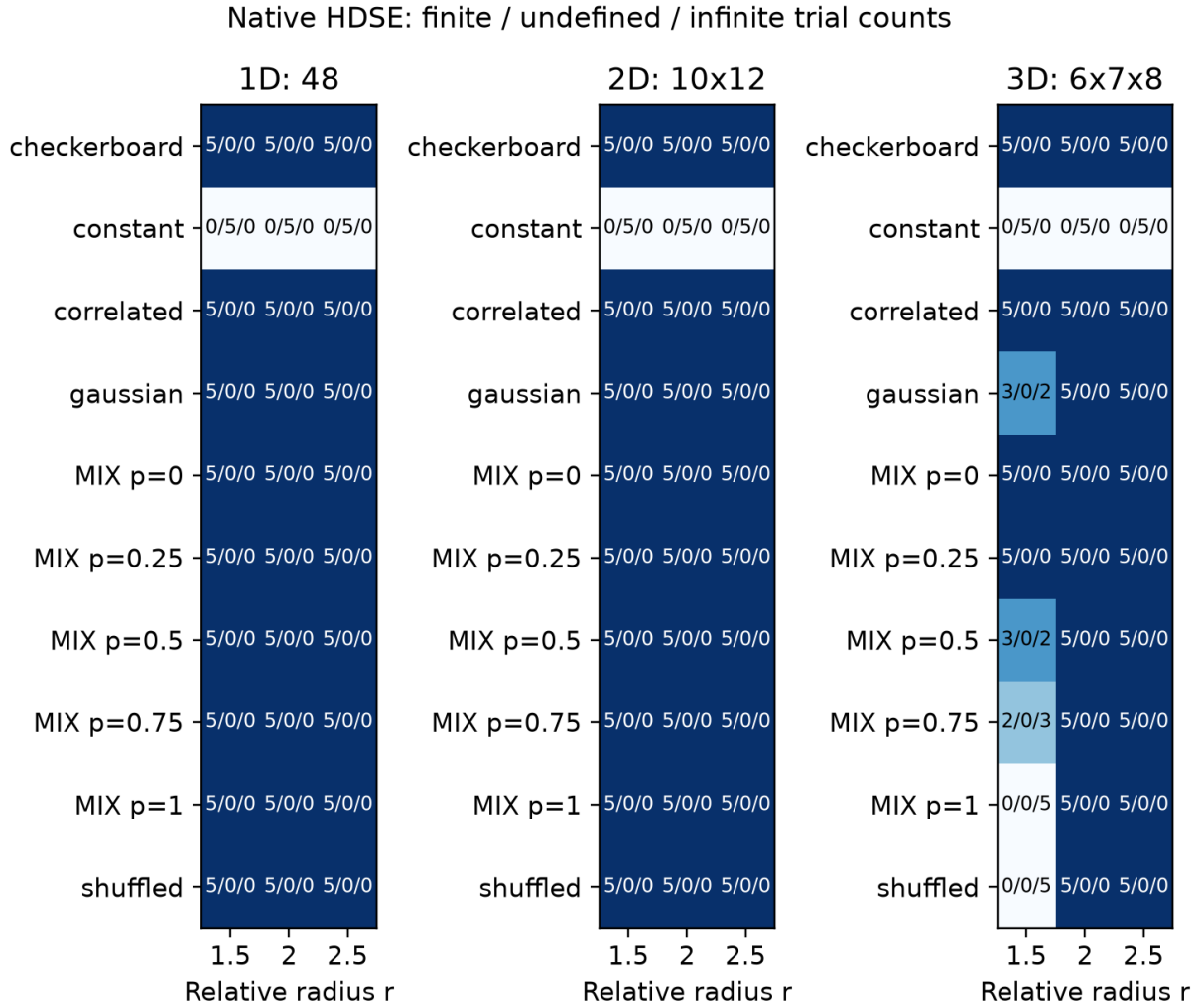


Figure 5: Finite, undefined and infinite outcomes across the replicated design. These statuses are retained rather than imputed as zero.

Matching evidence at $r=2$; trial means, not independent-pair probabilities

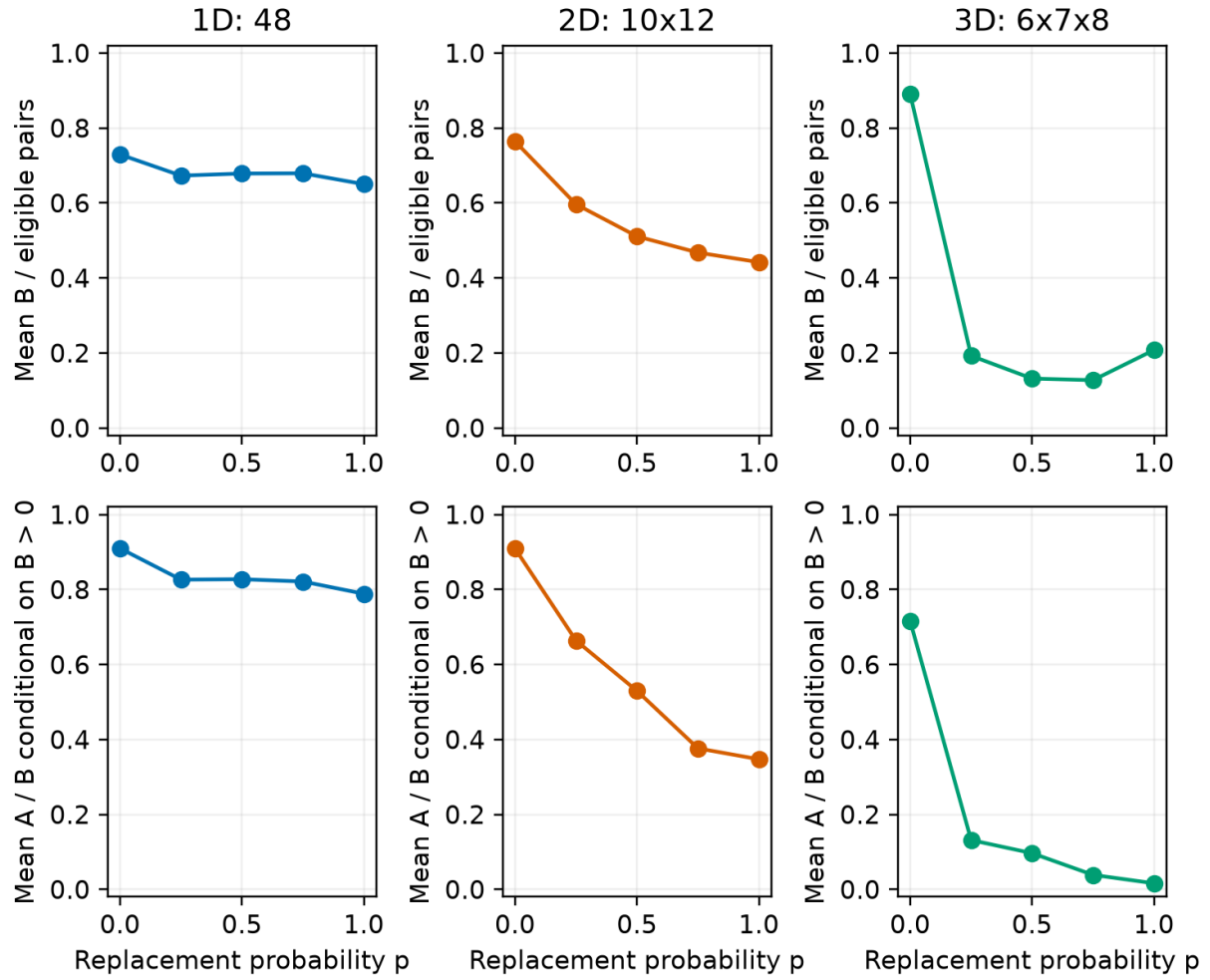


Figure 6: Base and extended template matching probabilities from observed pair counts. Sparse extended matches explain finite-sample degeneracy without reinterpreting it as zero entropy.

Shape	Control	r	Mean	SD	Finite/n	U/I
6x7x8	checkerboard	1.5	0	0	5/5	0/0
6x7x8	checkerboard	2	0	0	5/5	0/0
6x7x8	checkerboard	2.5	0	0	5/5	0/0
6x7x8	constant	1.5	-	-	0/5	5/0
6x7x8	constant	2	-	-	0/5	5/0
6x7x8	constant	2.5	-	-	0/5	5/0
6x7x8	correlated	1.5	2.56749	0.840728	5/5	0/0
6x7x8	correlated	2	1.37786	0.385547	5/5	0/0
6x7x8	correlated	2.5	0.753716	0.232607	5/5	0/0
6x7x8	gaussian	1.5	5.65724	0.495324	3/5	0/2
6x7x8	gaussian	2	3.3292	0.351848	5/5	0/0
6x7x8	gaussian	2.5	1.56662	0.232115	5/5	0/0
6x7x8	shuffled	1.5	-	-	0/5	0/5
6x7x8	shuffled	2	3.35688	0.471817	5/5	0/0
6x7x8	shuffled	2.5	1.76955	0.302802	5/5	0/0
10x12	checkerboard	1.5	0	0	5/5	0/0
10x12	checkerboard	2	0	0	5/5	0/0
10x12	checkerboard	2.5	0	0	5/5	0/0
10x12	constant	1.5	-	-	0/5	5/0
10x12	constant	2	-	-	0/5	5/0
10x12	constant	2.5	-	-	0/5	5/0
10x12	correlated	1.5	0.662661	0.109864	5/5	0/0
10x12	correlated	2	0.367965	0.0601135	5/5	0/0
10x12	correlated	2.5	0.185265	0.0324345	5/5	0/0
10x12	gaussian	1.5	1.74544	0.233531	5/5	0/0
10x12	gaussian	2	0.861448	0.128388	5/5	0/0
10x12	gaussian	2.5	0.400276	0.0465602	5/5	0/0
10x12	shuffled	1.5	1.79204	0.0637443	5/5	0/0
10x12	shuffled	2	0.850403	0.0587828	5/5	0/0
10x12	shuffled	2.5	0.409024	0.027914	5/5	0/0
48	checkerboard	1.5	0	0	5/5	0/0
48	checkerboard	2	0	0	5/5	0/0
48	checkerboard	2.5	0	0	5/5	0/0
48	constant	1.5	-	-	0/5	5/0
48	constant	2	-	-	0/5	5/0
48	constant	2.5	-	-	0/5	5/0
48	correlated	1.5	0.139492	0.0537352	5/5	0/0
48	correlated	2	0.0738259	0.0204956	5/5	0/0
48	correlated	2.5	0.0390499	0.0114501	5/5	0/0
48	gaussian	1.5	0.354011	0.0389952	5/5	0/0
48	gaussian	2	0.172839	0.026053	5/5	0/0
48	gaussian	2.5	0.0831918	0.0118893	5/5	0/0
48	shuffled	1.5	0.360859	0.0357175	5/5	0/0
48	shuffled	2	0.208973	0.018994	5/5	0/0
48	shuffled	2.5	0.0736555	0.00547319	5/5	0/0

The geometry comparison is read together with feature counts and populations. Neither a difference nor an agreement alone establishes estimator superiority. The observed tables are specific to the declared synthetic design, radius, finite sample and block geometry.

6.4 Related entropy and statistical complexity

The recorded normalization options make these related-measure summaries dimensionless. Display labels are DistEn: distribution entropy; PermEn: permutation; Spectral: spectral; Shannon: shannon; C-JS: statistical complexity.

Shape	p	Estimator	Finite mean	SD	Finite/n	U/I
6x7x8	0	DistEn	0.821272	0	5/5	0/0
6x7x8	0	PermEn	0.401562	0	5/5	0/0
6x7x8	0	Spectral	0.366254	0	5/5	0/0
6x7x8	0	Shannon	0.655986	0	5/5	0/0
6x7x8	0	C-JS	0.304078	0	5/5	0/0
6x7x8	0.25	DistEn	0.895451	0.00818597	5/5	0/0
6x7x8	0.25	PermEn	0.496046	0.00286137	5/5	0/0
6x7x8	0.25	Spectral	0.817511	0.0262881	5/5	0/0
6x7x8	0.25	Shannon	0.723031	0.00469153	5/5	0/0
6x7x8	0.25	C-JS	0.278564	0.00207836	5/5	0/0
6x7x8	0.5	DistEn	0.851999	0.00667078	5/5	0/0
6x7x8	0.5	PermEn	0.503431	0.000895235	5/5	0/0
6x7x8	0.5	Spectral	0.908797	0.0156529	5/5	0/0
6x7x8	0.5	Shannon	0.744093	0.00724335	5/5	0/0
6x7x8	0.5	C-JS	0.274515	0.00490519	5/5	0/0
6x7x8	0.75	DistEn	0.829532	0.00809795	5/5	0/0
6x7x8	0.75	PermEn	0.503852	0.000556784	5/5	0/0
6x7x8	0.75	Spectral	0.923803	0.00685935	5/5	0/0
6x7x8	0.75	Shannon	0.727915	0.00931895	5/5	0/0
6x7x8	0.75	C-JS	0.28245	0.00532371	5/5	0/0
6x7x8	1	DistEn	0.813712	0.00826181	5/5	0/0
6x7x8	1	PermEn	0.503478	0.000811645	5/5	0/0
6x7x8	1	Spectral	0.923819	0.00686466	5/5	0/0
6x7x8	1	Shannon	0.657322	0.00954592	5/5	0/0
6x7x8	1	C-JS	0.317542	0.00516416	5/5	0/0
10x12	0	DistEn	0.9211	0	5/5	0/0
10x12	0	PermEn	0.630036	0	5/5	0/0
10x12	0	Spectral	0.322933	0	5/5	0/0
10x12	0	Shannon	0.702175	0	5/5	0/0
10x12	0	C-JS	0.288771	0	5/5	0/0
10x12	0.25	DistEn	0.919696	0.0121944	5/5	0/0
10x12	0.25	PermEn	0.918879	0.0172518	5/5	0/0
10x12	0.25	Spectral	0.659491	0.0456326	5/5	0/0
10x12	0.25	Shannon	0.712075	0.00929196	5/5	0/0
10x12	0.25	C-JS	0.288816	0.00161396	5/5	0/0
10x12	0.5	DistEn	0.895524	0.0103245	5/5	0/0
10x12	0.5	PermEn	0.943309	0.0145696	5/5	0/0
10x12	0.5	Spectral	0.827952	0.0306717	5/5	0/0
10x12	0.5	Shannon	0.704415	0.00672185	5/5	0/0
10x12	0.5	C-JS	0.293462	0.00222378	5/5	0/0
10x12	0.75	DistEn	0.884852	0.00745327	5/5	0/0
10x12	0.75	PermEn	0.96377	0.00709848	5/5	0/0
10x12	0.75	Spectral	0.902698	0.0121331	5/5	0/0
10x12	0.75	Shannon	0.683502	0.0056531	5/5	0/0
10x12	0.75	C-JS	0.302502	0.00211954	5/5	0/0

Shape	p	Estimator	Finite mean	SD	Finite/n	U/I
10x12	1	DistEn	0.895472	0.00992491	5/5	0/0
10x12	1	PermEn	0.965971	0.00735781	5/5	0/0
10x12	1	Spectral	0.91021	0.0134747	5/5	0/0
10x12	1	Shannon	0.66164	0.00852925	5/5	0/0
10x12	1	C-JS	0.31495	0.00483384	5/5	0/0
48	0	DistEn	0.909918	0	5/5	0/0
48	0	PermEn	0.999673	0	5/5	0/0
48	0	Spectral	0.179052	0	5/5	0/0
48	0	Shannon	0.5	0	5/5	0/0
48	0	C-JS	0.331184	0	5/5	0/0
48	0.25	DistEn	0.915257	0.00752288	5/5	0/0
48	0.25	PermEn	0.993391	0.0035118	5/5	0/0
48	0.25	Spectral	0.777561	0.0341609	5/5	0/0
48	0.25	Shannon	0.578943	0.0354646	5/5	0/0
48	0.25	C-JS	0.325202	0.00666305	5/5	0/0
48	0.5	DistEn	0.929259	0.00851251	5/5	0/0
48	0.5	PermEn	0.986552	0.00983542	5/5	0/0
48	0.5	Spectral	0.845927	0.0531868	5/5	0/0
48	0.5	Shannon	0.613281	0.0207336	5/5	0/0
48	0.5	C-JS	0.322901	0.00458086	5/5	0/0
48	0.75	DistEn	0.948009	0.0133912	5/5	0/0
48	0.75	PermEn	0.993387	0.00574809	5/5	0/0
48	0.75	Spectral	0.882617	0.0293371	5/5	0/0
48	0.75	Shannon	0.629339	0.0270578	5/5	0/0
48	0.75	C-JS	0.320595	0.00573027	5/5	0/0
48	1	DistEn	0.958792	0.00697917	5/5	0/0
48	1	PermEn	0.999151	0.00116918	5/5	0/0
48	1	Spectral	0.880134	0.0274742	5/5	0/0
48	1	Shannon	0.63448	0.0272821	5/5	0/0
48	1	C-JS	0.318596	0.00497193	5/5	0/0

The entropy-complexity plane separates distributional uncertainty from distance to the uniform alphabet. A high normalized entropy and a high statistical complexity are different conditions. These fixed-amplitude-bin observations do not carry the causal ordinal interpretation of a Bandt-Pompe analysis.

Geometry changes template size and population ($r=2$)

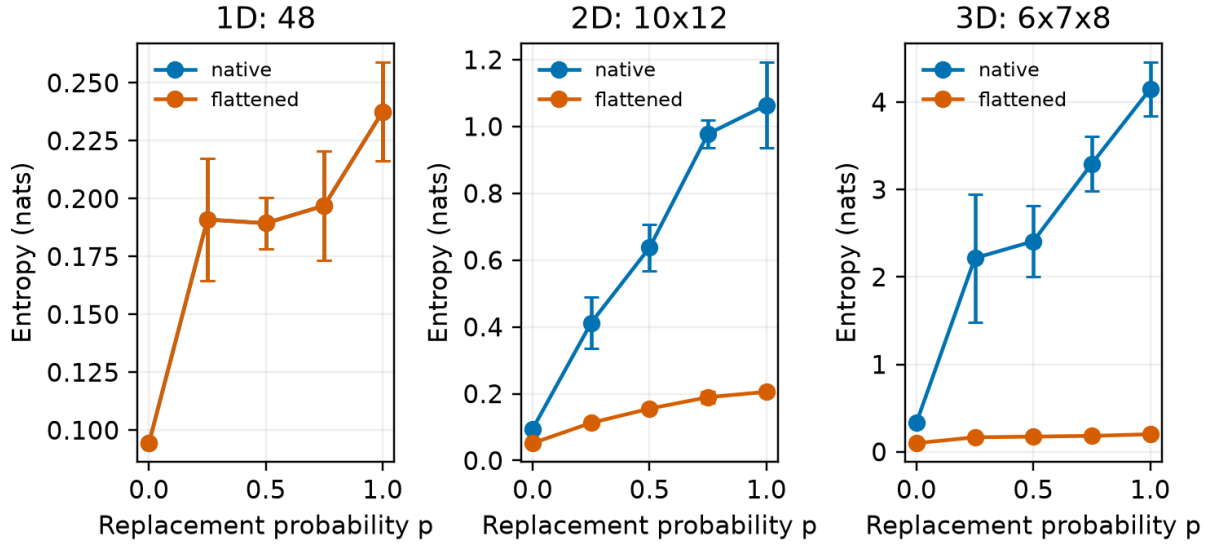


Figure 7: Native spatial HDSE and row-major flattened sample entropy. Their template feature counts and extension geometries differ; the comparison does not isolate flattening alone.

Fixed-alphabet entropy-complexity plane; every trial shown

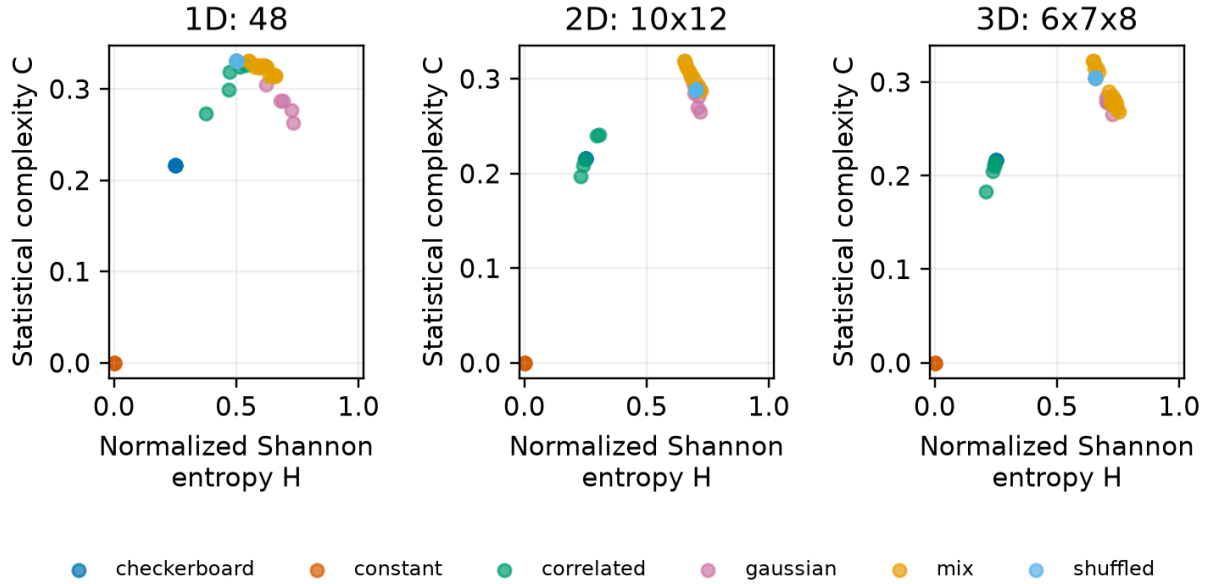


Figure 8: Normalized Shannon entropy and Jensen-Shannon statistical complexity of the complete configured fixed-bin amplitude alphabet. This is a descriptive distributional extension, not a causal ordinal classification.

7 Inference and prospective operating characteristics

7.1 Scientific unit and declared design

Power depends on a specified generating mechanism, estimand, comparison rule and decision threshold; it is not an intrinsic property of an entropy value. This simulation evaluates the complete configured procedure using independently generated whole fields [Morris et al., 2019]. Pixels and overlapping template pairs contribute to an estimator but do not become independent experimental subjects. A same-probability stochastic scenario provides the null control; alternative scenarios change the generating replacement probability.

The consumed design uses shapes 32, 8x10, 5x6x7, group sizes 4, 8, 16 and 64 independent outer repetitions per cell, with root seed 20261010. The comparison is independent and two_sided; the configured allocation mode is auto. The significance threshold is 0.05, interval confidence level 0.95 and configured planning target 0.8. Multiplicity policy: holm. The declared estimators are Native HDSE (r=2.5), Native HDSE (r=1.5), Normalized Shannon entropy, Statistical complexity.

Scenario	Role	First p	Second p
Identical distributions	null	0.25	0.25
Moderate replacement contrast	alternative	0.25	0.5
Large replacement contrast	alternative	0.25	0.75

Control	Configured value
schema version	1
seed	20261010
shapes	[[32], [8, 10], [5, 6, 7]]
sample sizes	[4, 8, 16]
repetitions	64
alpha	0.05
confidence level	0.95
target power	0.8
multiplicity	“holm”
period	12.0
phase origin	1
symbol range	[-4.0, 4.0]
symbol bins	16
max input elements	10000
max fields	20000
max distance component visits	5000000000
max inference work	100000000
max pool bytes	1048576
max report bytes	95000000

Inference control	Configured value
seed	20261011
pairing	“independent”
resamples	499
max samples	100
max resampling work	1000000

Inference control	Configured value
alternative	“two_sided”
mode	“auto”
max exact allocations	1000

7.1.1 Native HDSE (r=2.5)

Geometry: native; registered measure: `hdse`. Explicit invocation parameters:

```
{
  "embedding": {
    "max_memory_bytes": 134217728,
    "max_pairs": 500000,
    "max_templates": 1000,
    "shape": 2
  },
  "match": {
    "block_size": 64,
    "tolerance": 2.5
  }
}
```

7.1.2 Native HDSE (r=1.5)

Geometry: native; registered measure: `hdse`. Explicit invocation parameters:

```
{
  "embedding": {
    "max_memory_bytes": 134217728,
    "max_pairs": 500000,
    "max_templates": 1000,
    "shape": 2
  },
  "match": {
    "block_size": 64,
    "tolerance": 1.5
  }
}
```

7.1.3 Normalized Shannon entropy

Geometry: symbols; registered measure: `shannon_entropy`. Explicit invocation parameters:

```
{
  "normalize": true
}
```

7.1.4 Statistical complexity

Geometry: symbols; registered measure: `statistical_complexity`. Explicit invocation parameters:

```
{
  "config": {
```

```

    "alphabet_size": 16
  }
}

```

The report retains 1152 stored pools, 18432 whole-field estimator records and 6912 inference decisions: 6336 available and 576 unavailable. It emits 108 operating-characteristic cells. The preflight worst-case binomial Monte Carlo standard-error bound is 0.0625; it is a simulation-precision bound, not a statement of scientific effect precision.

n	Allocations	Evaluated	Mode	Work	Grid increment
4	70	70	exact	560	0.0142857
8	12870	499	monte_carlo	7984	0.002
16	601080390	499	monte_carlo	15968	0.002

For equal-sized groups with exact two-sided enumeration, every allocation has a distinct complementary allocation with the same absolute contrast. The minimum attainable p-value is at least twice the exact grid increment. Ties can increase it. The first Holm threshold is alpha divided by the declared family size; if even this lower bound exceeds that threshold, no family member can reject. This necessary-condition calculation does not guarantee rejection at other group sizes.

n	Two-sided p floor	First threshold	Rejection by floor
4	0.0285714	0.0125	precluded

Scheduled resource	Count
Fields	18432
Decisions	6912
Distance component visits	1971634176
Inference scalar work	56475648
Peak live pool arrays (bytes)	53760

Power plan: de602e182e541b4a64956b1dc30b661c29ac4f2f4b742a9695de245c80fb2914
Power report: 1e4ff2f8b8a38d3c29916d3d6f905ffb3ebbf240c2f0adfaa6526f2b1b1fb301

Sample-size comparisons reuse prefixes of each maximum-size subject pool. Within a given cell, outer repetitions use separate streams; estimates across sample sizes and estimators need not be independent. Reported intervals are pointwise rather than simultaneous confidence bands. The configured target is a planning input, not a certified sample-size recommendation or an estimate of post hoc observed power.

7.2 Permutation decisions and multiplicity

For independent field groups, the statistic is the second-group mean minus the first-group mean. The null assumption is exchangeability of the whole-field observations under identical generating distributions. The separate paired API uses within-pair contrasts and exchangeable sign flips; the current field generator supplies independent groups instead.

With exact enumeration, $p = e/A$ for e inclusive-tail allocations among all A allocations, including the observed allocation. With B sampled allocations, the Monte Carlo correction is $p = (b + 1)/(B + 1)$ [Phipson and Smyth, 2010]. Inclusive tails are compared using exact rational contrasts of the stored binary64 inputs; reporting a floating-point effect does not change the tail decision.

The recorded p-value grid increment need not equal the smallest attainable two-sided p-value, because complementary allocations can have the same absolute statistic.

When Holm adjustment is selected, ordered p-values use the running maximum of $(K - i + 1)p_{(i)}$, clipped at one and returned to the original order [Holm, 1979]. The declared family includes every configured estimator. Unavailable comparisons keep null observed p-values and cannot reject; a value of one is used only internally for conservative family bookkeeping. Raw and adjusted rejection counts remain separate in the producer report.

7.3 Availability, rejection rates and uncertainty

Let R be attempted outer repetitions, C available comparisons, $U = R - C$ unavailable comparisons and J selected rejections. The report distinguishes

$$\hat{q}_{\text{conditional}} = J/C \quad (C > 0), \quad \hat{q}_{\text{procedure}} = J/R, \quad \hat{a} = C/R. \quad (10)$$

Conditional rejection is undefined when no comparison is available. The unconditional rate describes the implemented procedure that declines to reject on unavailable comparisons. Under a null scenario it is a false-positive operating characteristic; under a declared alternative it is a prospective detection rate. Neither quantity is a biological classification accuracy.

Wilson score intervals quantify binomial Monte Carlo uncertainty at the configured confidence level [Brown et al., 2001]. If $\hat{q} = J/R$ and z is the configured normal quantile, their center and half-width are

$$c = \frac{\hat{q} + z^2/(2R)}{1 + z^2/R}, \quad h = \frac{z\sqrt{\hat{q}(1 - \hat{q})/R + z^2/(4R^2)}}{1 + z^2/R}. \quad (11)$$

The separate interval conditional on availability uses denominator C . Missing-decision identification bounds $[J/R, (J + U)/R]$ are sensitivity bounds, not confidence intervals. They show the range obtained by leaving unknown decisions unresolved rather than imputing zero entropy or deleting a field.

For a true cell rejection probability q , the outer simulation standard error is $\sqrt{q(1 - q)/R}$, bounded above by $1/(2\sqrt{R})$. This bound concerns simulation precision. It does not quantify an entropy estimator's bias or guarantee the configured power target is attained.

7.3.1 Identical distributions

Shape	n	Method	Available/R	Rejects/R	Unconditional [CI]
5x6x7	4	Native HDSE (r=2.5)	64/64	0/64	0 [0, 0.0566241]
5x6x7	4	Native HDSE (r=1.5)	0/64	0/64	0 [0, 0.0566241]
5x6x7	4	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
5x6x7	4	Statistical complexity	64/64	0/64	0 [0, 0.0566241]
5x6x7	8	Native HDSE (r=2.5)	64/64	4/64	0.0625 [0.0245712, 0.149975]

Shape	n	Method	Available/R	Rejects/R	Unconditional [CI]
5x6x7	8	Native HDSE ($r=1.5$)	0/64	0/64	0 [0, 0.0566241]
5x6x7	8	Normalized Shannon entropy	64/64	2/64	0.03125 [0.00861214, 0.106973]
5x6x7	8	Statistical complexity	64/64	2/64	0.03125 [0.00861214, 0.106973]
5x6x7	16	Native HDSE ($r=2.5$)	64/64	4/64	0.0625 [0.0245712, 0.149975]
5x6x7	16	Native HDSE ($r=1.5$)	0/64	0/64	0 [0, 0.0566241]
5x6x7	16	Normalized Shannon entropy	64/64	1/64	0.015625 [0.00276354, 0.083341]
5x6x7	16	Statistical complexity	64/64	1/64	0.015625 [0.00276354, 0.083341]
8x10	4	Native HDSE ($r=2.5$)	64/64	0/64	0 [0, 0.0566241]
8x10	4	Native HDSE ($r=1.5$)	64/64	0/64	0 [0, 0.0566241]
8x10	4	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
8x10	4	Statistical complexity	64/64	0/64	0 [0, 0.0566241]
8x10	8	Native HDSE ($r=2.5$)	64/64	0/64	0 [0, 0.0566241]
8x10	8	Native HDSE ($r=1.5$)	64/64	0/64	0 [0, 0.0566241]
8x10	8	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
8x10	8	Statistical complexity	64/64	0/64	0 [0, 0.0566241]
8x10	16	Native HDSE ($r=2.5$)	64/64	0/64	0 [0, 0.0566241]
8x10	16	Native HDSE ($r=1.5$)	64/64	2/64	0.03125 [0.00861214, 0.106973]
8x10	16	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
8x10	16	Statistical complexity	64/64	2/64	0.03125 [0.00861214, 0.106973]

Shape	n	Method	Available/R	Rejects/R	Unconditional [CI]
32	4	Native HDSE (r=2.5)	64/64	0/64	0 [0, 0.0566241]
32	4	Native HDSE (r=1.5)	64/64	0/64	0 [0, 0.0566241]
32	4	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
32	4	Statistical complexity	64/64	0/64	0 [0, 0.0566241]
32	8	Native HDSE (r=2.5)	64/64	2/64	0.03125 [0.00861214, 0.106973]
32	8	Native HDSE (r=1.5)	64/64	0/64	0 [0, 0.0566241]
32	8	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
32	8	Statistical complexity	64/64	0/64	0 [0, 0.0566241]
32	16	Native HDSE (r=2.5)	64/64	0/64	0 [0, 0.0566241]
32	16	Native HDSE (r=1.5)	64/64	1/64	0.015625 [0.00276354, 0.083341]
32	16	Normalized Shannon entropy	64/64	1/64	0.015625 [0.00276354, 0.083341]
32	16	Statistical complexity	64/64	0/64	0 [0, 0.0566241]

7.3.2 Moderate replacement contrast

Shape	n	Method	Available/R	Rejects/R	Unconditional [CI]
5x6x7	4	Native HDSE (r=2.5)	64/64	0/64	0 [0, 0.0566241]
5x6x7	4	Native HDSE (r=1.5)	0/64	0/64	0 [0, 0.0566241]
5x6x7	4	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
5x6x7	4	Statistical complexity	64/64	0/64	0 [0, 0.0566241]
5x6x7	8	Native HDSE (r=2.5)	64/64	18/64	0.28125 [0.185934, 0.401339]
5x6x7	8	Native HDSE (r=1.5)	0/64	0/64	0 [0, 0.0566241]

Shape	n	Method	Available/R	Rejects/R	Unconditional [CI]
5x6x7	8	Normalized Shannon entropy	64/64	64/64	1 [0.943376, 1]
5x6x7	8	Statistical complexity	64/64	63/64	0.984375 [0.916659, 0.997236]
5x6x7	16	Native HDSE (r=2.5)	64/64	33/64	0.515625 [0.395816, 0.633664]
5x6x7	16	Native HDSE (r=1.5)	0/64	0/64	0 [0, 0.0566241]
5x6x7	16	Normalized Shannon entropy	64/64	64/64	1 [0.943376, 1]
5x6x7	16	Statistical complexity	64/64	64/64	1 [0.943376, 1]
8x10	4	Native HDSE (r=2.5)	64/64	0/64	0 [0, 0.0566241]
8x10	4	Native HDSE (r=1.5)	64/64	0/64	0 [0, 0.0566241]
8x10	4	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
8x10	4	Statistical complexity	64/64	0/64	0 [0, 0.0566241]
8x10	8	Native HDSE (r=2.5)	64/64	47/64	0.734375 [0.615171, 0.827036]
8x10	8	Native HDSE (r=1.5)	64/64	58/64	0.90625 [0.810171, 0.956322]
8x10	8	Normalized Shannon entropy	64/64	3/64	0.046875 [0.016069, 0.128997]
8x10	8	Statistical complexity	64/64	12/64	0.1875 [0.110646, 0.299744]
8x10	16	Native HDSE (r=2.5)	64/64	62/64	0.96875 [0.893027, 0.991388]
8x10	16	Native HDSE (r=1.5)	64/64	63/64	0.984375 [0.916659, 0.997236]
8x10	16	Normalized Shannon entropy	64/64	3/64	0.046875 [0.016069, 0.128997]
8x10	16	Statistical complexity	64/64	31/64	0.484375 [0.366336, 0.604184]

Shape	n	Method	Available/R	Rejects/R	Unconditional [CI]
32	4	Native HDSE (r=2.5)	64/64	0/64	0 [0, 0.0566241]
32	4	Native HDSE (r=1.5)	64/64	0/64	0 [0, 0.0566241]
32	4	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
32	4	Statistical complexity	64/64	0/64	0 [0, 0.0566241]
32	8	Native HDSE (r=2.5)	64/64	2/64	0.03125 [0.00861214, 0.106973]
32	8	Native HDSE (r=1.5)	64/64	13/64	0.203125 [0.122734, 0.317136]
32	8	Normalized Shannon entropy	64/64	17/64	0.265625 [0.172964, 0.384829]
32	8	Statistical complexity	64/64	3/64	0.046875 [0.016069, 0.128997]
32	16	Native HDSE (r=2.5)	64/64	6/64	0.09375 [0.0436783, 0.189829]
32	16	Native HDSE (r=1.5)	64/64	42/64	0.65625 [0.534036, 0.760769]
32	16	Normalized Shannon entropy	64/64	41/64	0.640625 [0.518209, 0.747116]
32	16	Statistical complexity	64/64	7/64	0.109375 [0.0540011, 0.208986]

7.3.3 Large replacement contrast

Shape	n	Method	Available/R	Rejects/R	Unconditional [CI]
5x6x7	4	Native HDSE (r=2.5)	64/64	0/64	0 [0, 0.0566241]
5x6x7	4	Native HDSE (r=1.5)	0/64	0/64	0 [0, 0.0566241]
5x6x7	4	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
5x6x7	4	Statistical complexity	64/64	0/64	0 [0, 0.0566241]

Shape	n	Method	Available/R	Rejects/R	Unconditional [CI]
5x6x7	8	Native HDSE ($r=2.5$)	64/64	37/64	0.578125 [0.4561, 0.691302]
5x6x7	8	Native HDSE ($r=1.5$)	0/64	0/64	0 [0, 0.0566241]
5x6x7	8	Normalized Shannon entropy	64/64	64/64	1 [0.943376, 1]
5x6x7	8	Statistical complexity	64/64	48/64	0.75 [0.631837, 0.839851]
5x6x7	16	Native HDSE ($r=2.5$)	64/64	59/64	0.921875 [0.829805, 0.966169]
5x6x7	16	Native HDSE ($r=1.5$)	0/64	0/64	0 [0, 0.0566241]
5x6x7	16	Normalized Shannon entropy	64/64	64/64	1 [0.943376, 1]
5x6x7	16	Statistical complexity	64/64	64/64	1 [0.943376, 1]
8x10	4	Native HDSE ($r=2.5$)	64/64	0/64	0 [0, 0.0566241]
8x10	4	Native HDSE ($r=1.5$)	64/64	0/64	0 [0, 0.0566241]
8x10	4	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
8x10	4	Statistical complexity	64/64	0/64	0 [0, 0.0566241]
8x10	8	Native HDSE ($r=2.5$)	64/64	64/64	1 [0.943376, 1]
8x10	8	Native HDSE ($r=1.5$)	64/64	64/64	1 [0.943376, 1]
8x10	8	Normalized Shannon entropy	64/64	25/64	0.390625 [0.280556, 0.513081]
8x10	8	Statistical complexity	64/64	61/64	0.953125 [0.871003, 0.983931]
8x10	16	Native HDSE ($r=2.5$)	64/64	64/64	1 [0.943376, 1]
8x10	16	Native HDSE ($r=1.5$)	64/64	64/64	1 [0.943376, 1]
8x10	16	Normalized Shannon entropy	64/64	46/64	0.71875 [0.598661, 0.814066]
8x10	16	Statistical complexity	64/64	64/64	1 [0.943376, 1]

Shape	n	Method	Available/R	Rejects/R	Unconditional [CI]
32	4	Native HDSE ($r=2.5$)	64/64	0/64	0 [0, 0.0566241]
32	4	Native HDSE ($r=1.5$)	64/64	0/64	0 [0, 0.0566241]
32	4	Normalized Shannon entropy	64/64	0/64	0 [0, 0.0566241]
32	4	Statistical complexity	64/64	0/64	0 [0, 0.0566241]
32	8	Native HDSE ($r=2.5$)	64/64	11/64	0.171875 [0.0987774, 0.282132]
32	8	Native HDSE ($r=1.5$)	64/64	29/64	0.453125 [0.337294, 0.574264]
32	8	Normalized Shannon entropy	64/64	48/64	0.75 [0.631837, 0.839851]
32	8	Statistical complexity	64/64	8/64	0.125 [0.0647224, 0.227746]
32	16	Native HDSE ($r=2.5$)	64/64	20/64	0.3125 [0.212311, 0.433923]
32	16	Native HDSE ($r=1.5$)	64/64	51/64	0.796875 [0.682864, 0.877266]
32	16	Normalized Shannon entropy	64/64	63/64	0.984375 [0.916659, 0.997236]
32	16	Statistical complexity	64/64	22/64	0.34375 [0.239231, 0.465964]

Holm family; 95% pointwise Wilson intervals
Solid: all attempts; dotted: available comparisons

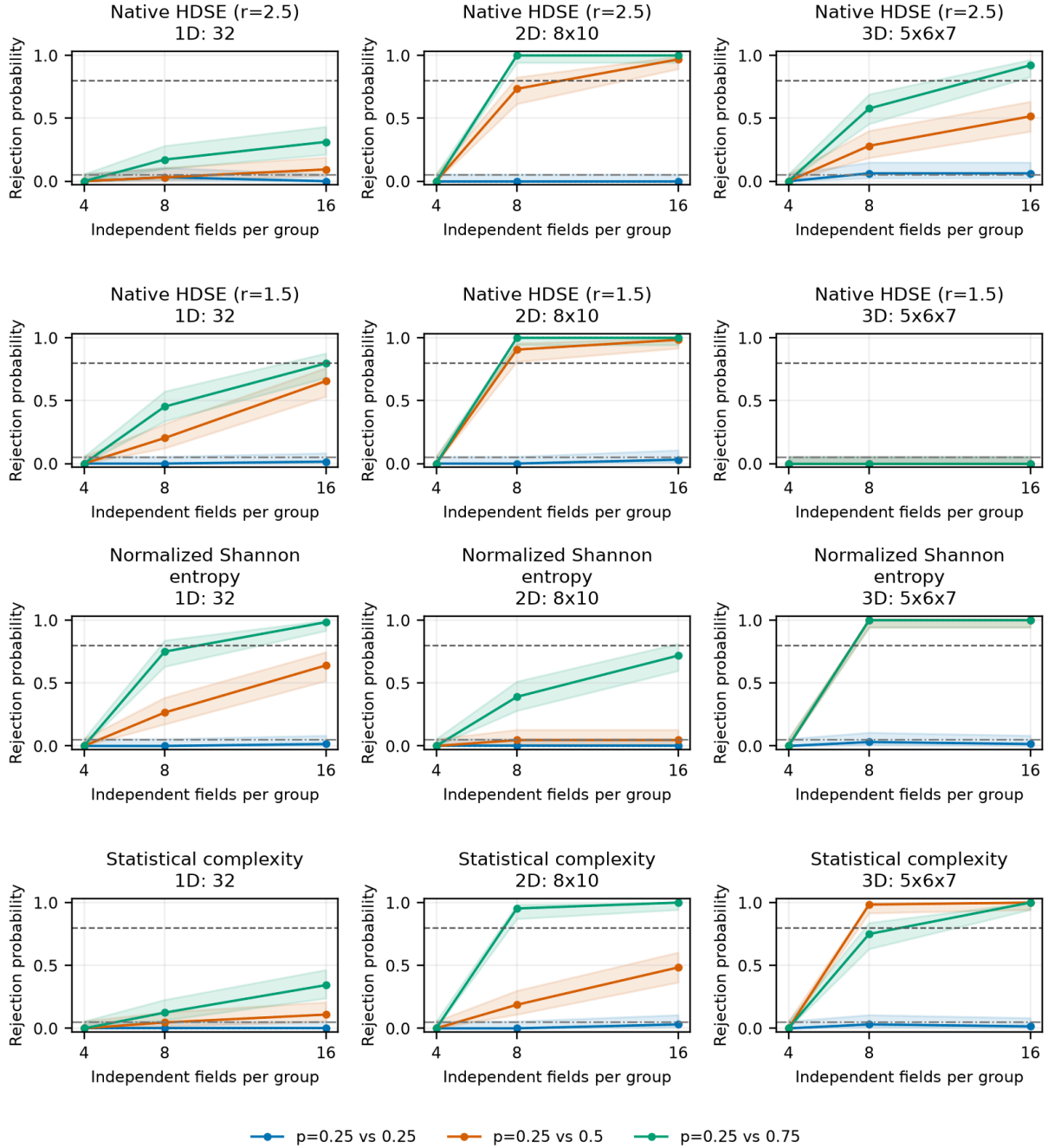


Figure 9: Prospective whole-field rejection rates at alpha 0.05, with holm adjustment over 4 declared estimators. Each cell uses 64 outer repetitions and group sizes 4, 8, 16. Solid curves count all attempts; dotted curves condition on availability, with gaps where no comparison is available. Shading shows pointwise Wilson intervals at confidence level 0.95. Horizontal references show the configured target 0.8 and alpha. Unavailable decisions remain unrejected in unconditional rates.

7.3.4 Identical distributions

Shape	n	Method	Availability [CI]	Conditional [CI]	Missing bounds
5x6x7	4	Native HDSE (r=2.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
5x6x7	4	Native HDSE (r=1.5)	0 [0, 0.0566241]	undefined	[0, 1]
5x6x7	4	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
5x6x7	4	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
5x6x7	8	Native HDSE (r=2.5)	1 [0.943376, 1]	0.0625 [0.0245712, 0.149975]	[0.0625, 0.0625]
5x6x7	8	Native HDSE (r=1.5)	0 [0, 0.0566241]	undefined	[0, 1]
5x6x7	8	Normalized Shannon entropy	1 [0.943376, 1]	0.03125 [0.00861214, 0.106973]	[0.03125, 0.03125]
5x6x7	8	Statistical complexity	1 [0.943376, 1]	0.03125 [0.00861214, 0.106973]	[0.03125, 0.03125]
5x6x7	16	Native HDSE (r=2.5)	1 [0.943376, 1]	0.0625 [0.0245712, 0.149975]	[0.0625, 0.0625]
5x6x7	16	Native HDSE (r=1.5)	0 [0, 0.0566241]	undefined	[0, 1]
5x6x7	16	Normalized Shannon entropy	1 [0.943376, 1]	0.015625 [0.00276354, 0.083341]	[0.015625, 0.015625]
5x6x7	16	Statistical complexity	1 [0.943376, 1]	0.015625 [0.00276354, 0.083341]	[0.015625, 0.015625]
8x10	4	Native HDSE (r=2.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	4	Native HDSE (r=1.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	4	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	4	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	8	Native HDSE (r=2.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	8	Native HDSE (r=1.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	8	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]

Shape	n	Method	Availability [CI]	Conditional [CI]	Missing bounds
8x10	8	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	16	Native HDSE ($r=2.5$)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	16	Native HDSE ($r=1.5$)	1 [0.943376, 1]	0.03125 [0.00861214, 0.106973]	[0.03125, 0.03125]
8x10	16	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	16	Statistical complexity	1 [0.943376, 1]	0.03125 [0.00861214, 0.106973]	[0.03125, 0.03125]
32	4	Native HDSE ($r=2.5$)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	4	Native HDSE ($r=1.5$)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	4	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	4	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	8	Native HDSE ($r=2.5$)	1 [0.943376, 1]	0.03125 [0.00861214, 0.106973]	[0.03125, 0.03125]
32	8	Native HDSE ($r=1.5$)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	8	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	8	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	16	Native HDSE ($r=2.5$)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	16	Native HDSE ($r=1.5$)	1 [0.943376, 1]	0.015625 [0.00276354, 0.083341]	[0.015625, 0.015625]
32	16	Normalized Shannon entropy	1 [0.943376, 1]	0.015625 [0.00276354, 0.083341]	[0.015625, 0.015625]
32	16	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]

7.3.5 Moderate replacement contrast

Shape	n	Method	Availability [CI]	Conditional [CI]	Missing bounds
5x6x7	4	Native HDSE (r=2.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
5x6x7	4	Native HDSE (r=1.5)	0 [0, 0.0566241]	undefined	[0, 1]
5x6x7	4	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
5x6x7	4	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
5x6x7	8	Native HDSE (r=2.5)	1 [0.943376, 1]	0.28125 [0.185934, 0.401339]	[0.28125, 0.28125]
5x6x7	8	Native HDSE (r=1.5)	0 [0, 0.0566241]	undefined	[0, 1]
5x6x7	8	Normalized Shannon entropy	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
5x6x7	8	Statistical complexity	1 [0.943376, 1]	0.984375 [0.916659, 0.997236]	[0.984375, 0.984375]
5x6x7	16	Native HDSE (r=2.5)	1 [0.943376, 1]	0.515625 [0.395816, 0.633664]	[0.515625, 0.515625]
5x6x7	16	Native HDSE (r=1.5)	0 [0, 0.0566241]	undefined	[0, 1]
5x6x7	16	Normalized Shannon entropy	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
5x6x7	16	Statistical complexity	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
8x10	4	Native HDSE (r=2.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	4	Native HDSE (r=1.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	4	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	4	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	8	Native HDSE (r=2.5)	1 [0.943376, 1]	0.734375 [0.615171, 0.827036]	[0.734375, 0.734375]
8x10	8	Native HDSE (r=1.5)	1 [0.943376, 1]	0.90625 [0.810171, 0.956322]	[0.90625, 0.90625]
8x10	8	Normalized Shannon entropy	1 [0.943376, 1]	0.046875 [0.016069, 0.128997]	[0.046875, 0.046875]

Shape	n	Method	Availability [CI]	Conditional [CI]	Missing bounds
8x10	8	Statistical complexity	1 [0.943376, 1]	0.1875 [0.110646, 0.299744]	[0.1875, 0.1875]
8x10	16	Native HDSE (r=2.5)	1 [0.943376, 1]	0.96875 [0.893027, 0.991388]	[0.96875, 0.96875]
8x10	16	Native HDSE (r=1.5)	1 [0.943376, 1]	0.984375 [0.916659, 0.997236]	[0.984375, 0.984375]
8x10	16	Normalized Shannon entropy	1 [0.943376, 1]	0.046875 [0.016069, 0.128997]	[0.046875, 0.046875]
8x10	16	Statistical complexity	1 [0.943376, 1]	0.484375 [0.366336, 0.604184]	[0.484375, 0.484375]
32	4	Native HDSE (r=2.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	4	Native HDSE (r=1.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	4	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	4	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	8	Native HDSE (r=2.5)	1 [0.943376, 1]	0.03125 [0.00861214, 0.106973]	[0.03125, 0.03125]
32	8	Native HDSE (r=1.5)	1 [0.943376, 1]	0.203125 [0.122734, 0.317136]	[0.203125, 0.203125]
32	8	Normalized Shannon entropy	1 [0.943376, 1]	0.265625 [0.172964, 0.384829]	[0.265625, 0.265625]
32	8	Statistical complexity	1 [0.943376, 1]	0.046875 [0.016069, 0.128997]	[0.046875, 0.046875]
32	16	Native HDSE (r=2.5)	1 [0.943376, 1]	0.09375 [0.0436783, 0.189829]	[0.09375, 0.09375]
32	16	Native HDSE (r=1.5)	1 [0.943376, 1]	0.65625 [0.534036, 0.760769]	[0.65625, 0.65625]
32	16	Normalized Shannon entropy	1 [0.943376, 1]	0.640625 [0.518209, 0.747116]	[0.640625, 0.640625]
32	16	Statistical complexity	1 [0.943376, 1]	0.109375 [0.0540011, 0.208986]	[0.109375, 0.109375]

7.3.6 Large replacement contrast

Shape	n	Method	Availability [CI]	Conditional [CI]	Missing bounds
5x6x7	4	Native HDSE ($r=2.5$)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
5x6x7	4	Native HDSE ($r=1.5$)	0 [0, 0.0566241]	undefined	[0, 1]
5x6x7	4	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
5x6x7	4	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
5x6x7	8	Native HDSE ($r=2.5$)	1 [0.943376, 1]	0.578125 [0.4561, 0.691302]	[0.578125, 0.578125]
5x6x7	8	Native HDSE ($r=1.5$)	0 [0, 0.0566241]	undefined	[0, 1]
5x6x7	8	Normalized Shannon entropy	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
5x6x7	8	Statistical complexity	1 [0.943376, 1]	0.75 [0.631837, 0.839851]	[0.75, 0.75]
5x6x7	16	Native HDSE ($r=2.5$)	1 [0.943376, 1]	0.921875 [0.829805, 0.966169]	[0.921875, 0.921875]
5x6x7	16	Native HDSE ($r=1.5$)	0 [0, 0.0566241]	undefined	[0, 1]
5x6x7	16	Normalized Shannon entropy	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
5x6x7	16	Statistical complexity	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
8x10	4	Native HDSE ($r=2.5$)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	4	Native HDSE ($r=1.5$)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	4	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	4	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
8x10	8	Native HDSE ($r=2.5$)	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
8x10	8	Native HDSE ($r=1.5$)	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
8x10	8	Normalized Shannon entropy	1 [0.943376, 1]	0.390625 [0.280556, 0.513081]	[0.390625, 0.390625]

Shape	n	Method	Availability [CI]	Conditional [CI]	Missing bounds
8x10	8	Statistical complexity	1 [0.943376, 1]	0.953125 [0.871003, 0.983931]	[0.953125, 0.953125]
8x10	16	Native HDSE (r=2.5)	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
8x10	16	Native HDSE (r=1.5)	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
8x10	16	Normalized Shannon entropy	1 [0.943376, 1]	0.71875 [0.598661, 0.814066]	[0.71875, 0.71875]
8x10	16	Statistical complexity	1 [0.943376, 1]	1 [0.943376, 1]	[1, 1]
32	4	Native HDSE (r=2.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	4	Native HDSE (r=1.5)	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	4	Normalized Shannon entropy	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	4	Statistical complexity	1 [0.943376, 1]	0 [0, 0.0566241]	[0, 0]
32	8	Native HDSE (r=2.5)	1 [0.943376, 1]	0.171875 [0.0987774, 0.282132]	[0.171875, 0.171875]
32	8	Native HDSE (r=1.5)	1 [0.943376, 1]	0.453125 [0.337294, 0.574264]	[0.453125, 0.453125]
32	8	Normalized Shannon entropy	1 [0.943376, 1]	0.75 [0.631837, 0.839851]	[0.75, 0.75]
32	8	Statistical complexity	1 [0.943376, 1]	0.125 [0.0647224, 0.227746]	[0.125, 0.125]
32	16	Native HDSE (r=2.5)	1 [0.943376, 1]	0.3125 [0.212311, 0.433923]	[0.3125, 0.3125]
32	16	Native HDSE (r=1.5)	1 [0.943376, 1]	0.796875 [0.682864, 0.877266]	[0.796875, 0.796875]
32	16	Normalized Shannon entropy	1 [0.943376, 1]	0.984375 [0.916659, 0.997236]	[0.984375, 0.984375]
32	16	Statistical complexity	1 [0.943376, 1]	0.34375 [0.239231, 0.465964]	[0.34375, 0.34375]

Stochastic null control; 95% pointwise Wilson intervals
Bottom: solid holm; hollow squares unadjusted

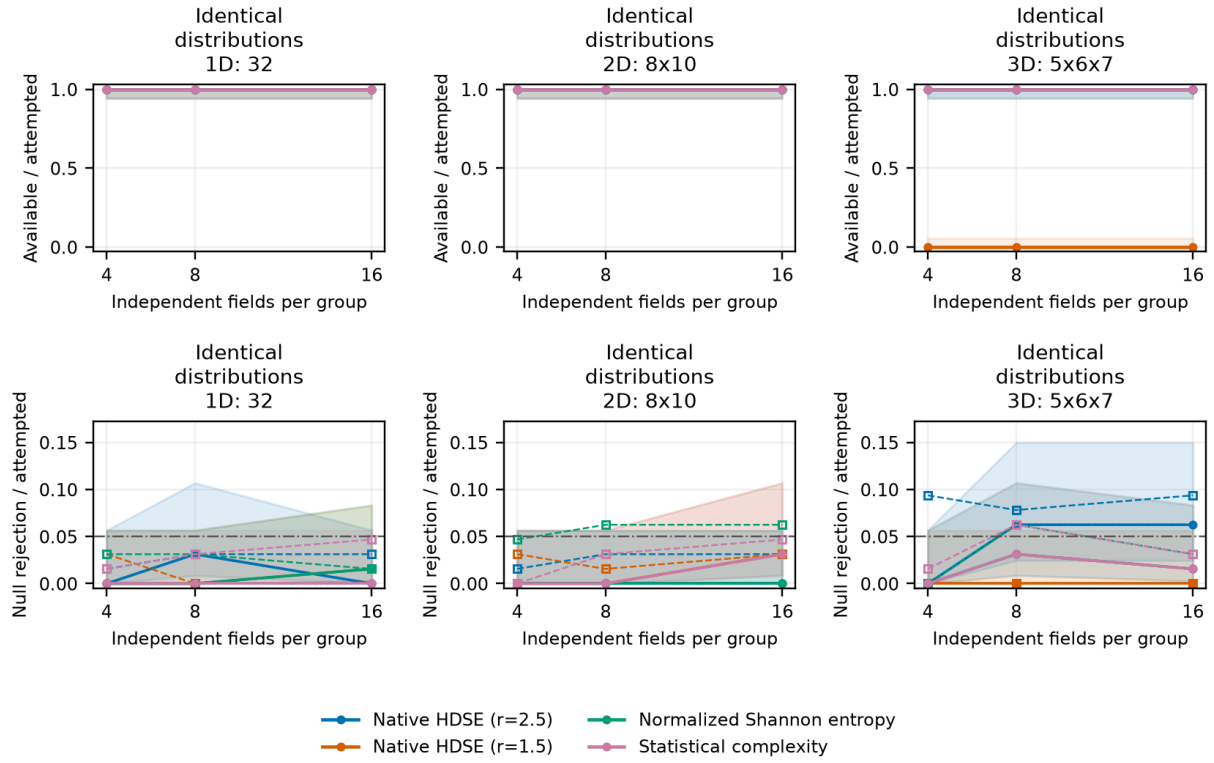


Figure 10: Availability and unconditional null calibration for Identical distributions ($p=0.25$ in both groups). Raw rejection is compared with holm adjustment over 4 declared estimators at alpha 0.05. Whole fields are the independent units; 64 repetitions estimate each cell. Intervals are pointwise at confidence level 0.95.

7.4 Effect summaries and bootstrap interface

Mean differences retain each estimator's configured units. Their reported means and sample standard deviations condition on available comparisons; unavailable effects remain null. Empirical variation among outer repetitions and the Monte Carlo uncertainty of a rejection proportion are different quantities.

7.4.1 Identical distributions

Shape	n	Method	Mean Δ	SD	Raw	Adj.
5x6x7	4	Native HDSE ($r=2.5$)	0.0079771	0.499975	6	0
5x6x7	4	Native HDSE ($r=1.5$)	-	-	0	0
5x6x7	4	Normalized Shannon entropy	0.000465629	0.00874333	0	0
5x6x7	4	Statistical complexity	8.73969e-05	0.00293578	1	0
5x6x7	8	Native HDSE ($r=2.5$)	0.0110846	0.3332	5	4
5x6x7	8	Native HDSE ($r=1.5$)	-	-	0	0
5x6x7	8	Normalized Shannon entropy	-0.000363409	0.00665451	4	2
5x6x7	8	Statistical complexity	0.000327146	0.00232198	4	2
5x6x7	16	Native HDSE ($r=2.5$)	0.00720238	0.233063	6	4
5x6x7	16	Native HDSE ($r=1.5$)	-	-	0	0
5x6x7	16	Normalized Shannon entropy	-0.000233572	0.00485811	2	1
5x6x7	16	Statistical complexity	0.000267878	0.00160841	2	1
8x10	4	Native HDSE ($r=2.5$)	0.000128229	0.0319748	1	0
8x10	4	Native HDSE ($r=1.5$)	0.0128598	0.162703	2	0
8x10	4	Normalized Shannon entropy	-0.00027704	0.0113081	3	0
8x10	4	Statistical complexity	-0.000342759	0.00218425	0	0
8x10	8	Native HDSE ($r=2.5$)	0.000280026	0.0233939	2	0
8x10	8	Native HDSE ($r=1.5$)	0.00905552	0.106368	1	0
8x10	8	Normalized Shannon entropy	0.000518915	0.00762356	4	0
8x10	8	Statistical complexity	-0.000204289	0.00151584	2	0
8x10	16	Native HDSE ($r=2.5$)	0.00341973	0.0179267	2	0
8x10	16	Native HDSE ($r=1.5$)	0.013453	0.0848148	2	2
8x10	16	Normalized Shannon entropy	0.000831496	0.00487999	4	0
8x10	16	Statistical complexity	-0.000162677	0.00112663	3	2

Shape	n	Method	Mean Δ	SD	Raw	Adj.
32	4	Native HDSE ($r=2.5$)	0.00119658	0.011672	1	0
32	4	Native HDSE ($r=1.5$)	0.00980362	0.0355917	2	0
32	4	Normalized Shannon entropy	-0.00112075	0.0209735	2	0
32	4	Statistical complexity	0.000143083	0.00264442	1	0
32	8	Native HDSE ($r=2.5$)	0.00123023	0.00742757	2	2
32	8	Native HDSE ($r=1.5$)	0.00533874	0.0222971	0	0
32	8	Normalized Shannon entropy	0.00014562	0.013859	2	0
32	8	Statistical complexity	0.000185304	0.00164353	2	0
32	16	Native HDSE ($r=2.5$)	0.000501501	0.0053336	2	0
32	16	Native HDSE ($r=1.5$)	0.00437018	0.0208222	1	1
32	16	Normalized Shannon entropy	0.000254737	0.0108428	1	1
32	16	Statistical complexity	-5.40386e-05	0.00141895	3	0

7.4.2 Moderate replacement contrast

Shape	n	Method	Mean Δ	SD	Raw	Adj.
5x6x7	4	Native HDSE ($r=2.5$)	-0.401807	0.313747	7	0
5x6x7	4	Native HDSE ($r=1.5$)	-	-	0	0
5x6x7	4	Normalized Shannon entropy	0.0566313	0.0104234	63	0
5x6x7	4	Statistical complexity	-0.0137403	0.00409393	45	0
5x6x7	8	Native HDSE ($r=2.5$)	-0.480408	0.254119	28	18
5x6x7	8	Native HDSE ($r=1.5$)	-	-	0	0
5x6x7	8	Normalized Shannon entropy	0.055641	0.00661119	64	64
5x6x7	8	Statistical complexity	-0.0133952	0.00265866	63	63
5x6x7	16	Native HDSE ($r=2.5$)	-0.452019	0.177469	42	33
5x6x7	16	Native HDSE ($r=1.5$)	-	-	0	0
5x6x7	16	Normalized Shannon entropy	0.0555227	0.00448515	64	64
5x6x7	16	Statistical complexity	-0.0134184	0.00210074	64	64
8x10	4	Native HDSE ($r=2.5$)	0.100478	0.0396488	23	0

Shape	n	Method	Mean Δ	SD	Raw	Adj.
8x10	4	Native HDSE ($r=1.5$)	0.501634	0.184456	26	0
8x10	4	Normalized Shannon entropy	-0.00193156	0.00912914	1	0
8x10	4	Statistical complexity	0.00386839	0.00275411	3	0
8x10	8	Native HDSE ($r=2.5$)	0.102142	0.0226454	58	47
8x10	8	Native HDSE ($r=1.5$)	0.525782	0.103121	63	58
8x10	8	Normalized Shannon entropy	-0.00040937	0.00729443	4	3
8x10	8	Statistical complexity	0.00341839	0.00211569	19	12
8x10	16	Native HDSE ($r=2.5$)	0.100598	0.0193475	64	62
8x10	16	Native HDSE ($r=1.5$)	0.535309	0.0839673	64	63
8x10	16	Normalized Shannon entropy	-0.00125023	0.00489409	4	3
8x10	16	Statistical complexity	0.00337688	0.00143883	35	31
32	4	Native HDSE ($r=2.5$)	0.00746029	0.0114391	4	0
32	4	Native HDSE ($r=1.5$)	0.0608876	0.0433475	6	0
32	4	Normalized Shannon entropy	0.0337575	0.0201683	9	0
32	4	Statistical complexity	-0.00217528	0.00308082	2	0
32	8	Native HDSE ($r=2.5$)	0.00611197	0.00785284	4	2
32	8	Native HDSE ($r=1.5$)	0.0624163	0.0280534	28	13
32	8	Normalized Shannon entropy	0.0343612	0.0163264	37	17
32	8	Statistical complexity	-0.0019657	0.00218381	9	3
32	16	Native HDSE ($r=2.5$)	0.00728931	0.00582401	13	6
32	16	Native HDSE ($r=1.5$)	0.0620799	0.0228046	48	42
32	16	Normalized Shannon entropy	0.0364256	0.0124558	53	41
32	16	Statistical complexity	-0.00236393	0.00155549	10	7

7.4.3 Large replacement contrast

Shape	n	Method	Mean Δ	SD	Raw	Adj.
5x6x7	4	Native HDSE ($r=2.5$)	-0.621122	0.344539	18	0
5x6x7	4	Native HDSE ($r=1.5$)	-	-	0	0

Shape	n	Method	Mean Δ	SD	Raw	Adj.
5x6x7	4	Normalized Shannon entropy	0.0551501	0.00700463	64	0
5x6x7	4	Statistical complexity	-0.010481	0.00384784	29	0
5x6x7	8	Native HDSE (r=2.5)	-0.643889	0.233606	44	37
5x6x7	8	Native HDSE (r=1.5)	-	-	0	0
5x6x7	8	Normalized Shannon entropy	0.0558269	0.00543524	64	64
5x6x7	8	Statistical complexity	-0.0107382	0.00278446	56	48
5x6x7	16	Native HDSE (r=2.5)	-0.642828	0.166691	62	59
5x6x7	16	Native HDSE (r=1.5)	-	-	0	0
5x6x7	16	Normalized Shannon entropy	0.0548904	0.00426106	64	64
5x6x7	16	Statistical complexity	-0.0101608	0.00206099	64	64
8x10	4	Native HDSE (r=2.5)	0.174859	0.0510583	48	0
8x10	4	Native HDSE (r=1.5)	0.982899	0.199453	61	0
8x10	4	Normalized Shannon entropy	-0.0136454	0.0107619	9	0
8x10	4	Statistical complexity	0.0105099	0.00384207	37	0
8x10	8	Native HDSE (r=2.5)	0.177121	0.031014	64	64
8x10	8	Native HDSE (r=1.5)	0.984111	0.143202	64	64
8x10	8	Normalized Shannon entropy	-0.0145816	0.00796583	25	25
8x10	8	Statistical complexity	0.0111532	0.00294683	63	61
8x10	16	Native HDSE (r=2.5)	0.178323	0.0236021	64	64
8x10	16	Native HDSE (r=1.5)	0.989584	0.104276	64	64
8x10	16	Normalized Shannon entropy	-0.0141923	0.00524524	46	46
8x10	16	Statistical complexity	0.0109956	0.00187163	64	64
32	4	Native HDSE (r=2.5)	0.00989782	0.0128064	5	0
32	4	Native HDSE (r=1.5)	0.0777537	0.0403601	15	0
32	4	Normalized Shannon entropy	0.0537122	0.0211495	25	0
32	4	Statistical complexity	-0.00268643	0.00338277	5	0
32	8	Native HDSE (r=2.5)	0.0106385	0.00956086	15	11
32	8	Native HDSE (r=1.5)	0.0783142	0.0275142	43	29

Shape	n	Method	Mean Δ	SD	Raw	Adj.
32	8	Normalized Shannon entropy	0.0527435	0.0153472	58	48
32	8	Statistical complexity	-0.00310831	0.00249848	10	8
32	16	Native HDSE ($r=2.5$)	0.0107698	0.00652311	27	20
32	16	Native HDSE ($r=1.5$)	0.0767366	0.0215585	59	51
32	16	Normalized Shannon entropy	0.0543095	0.0120494	64	63
32	16	Statistical complexity	-0.00335883	0.00178672	27	22

The public percentile-bootstrap interface resamples whole observations within independent groups, or paired observations jointly, and returns configured empirical-quantile bounds [Efron, 1979]. It is a separate inference tool; the operating-characteristic simulation uses permutation decisions and Wilson intervals. For bootstrap contrasts $D^* = \bar{Y}^* - \bar{X}^*$ and configured confidence level c , the percentile limits are $[Q_{(1-c)/2}(D^*), Q_{(1+c)/2}(D^*)]$, using linear interpolation of empirical quantiles. The paired variant resamples subject pairs together and retains their within-pair dependence. Small, discrete, sparse and degenerate samples can limit percentile coverage. Application-specific dependence, effect size, generating assumptions and external validation remain necessary before choosing a scientific sample size or transferring the synthetic results to ant behavior.

8 Reproducibility and software architecture

8.1 Numerical and file boundaries

The numerical package separates array validation, geometry, blocked pair work and estimator-specific semantics. It does not import manuscript or plotting tooling. Immutable configurations reject unknown options. The registry binds the complete call signature, distinguishes observations from probability alphabets and retains effective parameters in every result. NPY loading disables pickle; CSV inputs retain their declared grid meaning. Strict JSON represents nonfinite estimates through explicit statuses and null values.

Tests use analytical examples, independently computed loop oracles, seeded arrays, real files and actual subprocesses. Deliberately invalid inputs and coherently altered artifact records are controls proving that checks can fail. Configured coverage floors are 90 percent for numerical source and 60 percent for research infrastructure.

The configured verification receipts report 904 passing tests, 94.5148 percent source coverage and 90.3481 percent infrastructure coverage. These are recorded runs, not a substitute for independent release review.

Receipt identities (SHA-256):

```
Source coverage: c0244352e54ccba78b5b6e9d0fbcead948e8d2dc95047f74e1d983792d0784d
Infra coverage:  ac676b0754db6e79de1fb87ab59963028dc2e35fd734412b096e789ce007ef8b
Test log:        234b15d68f951ea5ba9fc70d1078540dc38eeb67a66524c4d1d33ef29d84f46f
```

8.2 Source-bound manuscript generation

The title, author metadata, version, design, measurements, tables and figure paths are sourced from the validated publication profile, project manifest and generated reports. Template variables

must all resolve before rendering. The shared implementation is [docxology/template](https://github.com/docxology/template), pinned at `d39f2478da2512b42ad961e98a5f3b854723f92e`, using the canonical `projects/templates/template_code_project/manuscript` conventions. Actual installed Python and Lua source files are authenticated against their installer RECORD hashes. Pandoc, Pandoc-crossref, BibTeX and the selected LaTeX engine produce mathematical typesetting, resolved references and a real TOC. No custom ReportLab manuscript fallback is used.

The inspectable resolved section tree, variable inventory, renderer source/tool receipt, generated TeX/TOC diagnostics and final PDF remain distinct artifacts. Figure staging uses exact-byte copies of producer outputs because the shared PDF renderer expects a sibling figure directory. Staging does not recompute or alter those figures.

8.3 Provenance bindings

```
Results CSV SHA-256: a4104429035874774f07cb2ec816a1cd656504435f245754d730e3bc06526727
Study CSV SHA-256: 493fd105d2410a1ac66c3bcf75e2a0313dc69e7b6ccd58625508454cc3ae57ce
Study report SHA-256: e77af9a7e5a55ddca73d9f779e55cf5e715d60e77b0fadd7c93f91f409f425be
```

The manifest binds actual input, configuration, code and output bytes. Stored file, canonical array, configuration, result and table identities remain exact. Synthetic design replay alone uses a declared, machine-epsilon-derived float64 envelope after exact shape and finite-value checks. This accommodates bounded platform roundoff without changing stored inputs, smoothing matching thresholds or relabeling stale results. The initial hosted failure and its adversarial regression controls are retained in the project record.

The supplied published PDF is not redistributed. Journal acceptance, empirical source-paper replication and independent application validation are not established by the software build. Independent scientific and release review remains separate from these executable engineering checks.

9 Discussion and conclusions

Native geometry, input meaning and estimator convention are parts of a complexity analysis rather than preprocessing details that can be silently discarded. The framework makes those choices configurable and records their numerical consequences with counts and explicit statuses. The replicated study and computational records allow readers to examine tolerance, spatial representation, degeneracy and resource use together.

Finite conditional summaries must be interpreted alongside all outcome counts. The high-dimensional template feature population and finite origin geometry change together, while the native-versus-flattened comparison also changes continuation length. Stable ordinal ties, amplitude binning, spectral preprocessing and singular-value normalization are documented conventions. They remain application choices rather than universal definitions of complexity.

Exhaustive matching is quadratic in templates. Blocking bounds workspace, and the analytical estimator exposes the work schedule, but neither removes that growth. Host benchmarks describe observed execution on fixed inputs. New applications may require different data sizes, symbolization, boundary treatment or more efficient exact and approximate search methods, each with a new validation contract.

Prospective rejection depends on the complete generating and decision procedure. Whole-field permutation inference separates stochastic null calibration from declared alternative detection. Availability determines whether an estimator can support a comparison; conditional rejection alone can select a different population. The retained uncertainty intervals, missing-decision bounds and

allocation-resolution limits make that dependence inspectable. They support application-specific planning without certifying a sample size for biological measurements.

Ant collective behavior and other spatial scientific applications are future empirical domains, not findings of this synthetic study. Progress in those domains requires controlled measurements, appropriate null models, sensitivity analysis and independent review. The present contribution is a reproducible, typed methods implementation and source-bound research workflow through which those questions can be investigated.

References

Method definitions are attributed to their primary sources. The HDSE source, the local-template families, probability information, ordinal/decomposition methods and statistical-complexity family have separate citation records. Bibliographic entries are resolved by the shared template’s native bibliography pipeline; source metadata do not import the template exemplar’s publication DOI.

Christoph Bandt and Bernd Pompe. Permutation entropy: A natural complexity measure for time series. *Physical Review Letters*, 88:174102, 2002. doi: 10.1103/PhysRevLett.88.174102.

D. S. Broomhead and G. P. King. Extracting qualitative dynamics from experimental data. *Physica D: Nonlinear Phenomena*, 20:217–236, 1986. doi: 10.1016/0167-2789(86)90031-X.

Lawrence D. Brown, T. Tony Cai, and Anirban DasGupta. Interval estimation for a binomial proportion. *Statistical Science*, 16(2), 2001. doi: 10.1214/ss/1009213286.

Weiting Chen, Zhizhong Wang, Hongbo Xie, and Wangxin Yu. Characterization of surface emg signal based on fuzzy entropy. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 15:266–272, 2007. doi: 10.1109/TNSRE.2007.897025.

Madalena Costa, Ary L. Goldberger, and C.-K. Peng. Multiscale entropy analysis of complex physiologic time series. *Physical Review Letters*, 89:068102, 2002. doi: 10.1103/PhysRevLett.89.068102.

Bradley Efron. Bootstrap methods: Another look at the jackknife. *The Annals of Statistics*, 7(1), 1979. doi: 10.1214/aos/1176344552.

Bilal Fadlallah, Badong Chen, Andreas Keil, and Jose Principe. Weighted-permutation entropy: A complexity measure for time series incorporating amplitude information. *Physical Review E*, 87:022911, 2013. doi: 10.1103/PhysRevE.87.022911.

Sture Holm. A simple sequentially rejective multiple test procedure. *Scandinavian Journal of Statistics*, 6(2):65–70, 1979. URL <https://www.jstor.org/stable/4615733>.

Peng Li, Chengyu Liu, Ke Li, Dingchang Zheng, Changchun Liu, and Yinglong Hou. Assessing the complexity of short-term heartbeat interval series by distribution entropy. *Medical and Biological Engineering and Computing*, 53:77–87, 2015. doi: 10.1007/s11517-014-1216-0.

Jianhua Lin. Divergence measures based on the shannon entropy. *IEEE Transactions on Information Theory*, 37:145–151, 1991. doi: 10.1109/18.61115.

M. T. Martin, A. Plastino, and O. A. Rosso. Generalized statistical complexity measures: Geometrical and analytical properties. *Physica A: Statistical Mechanics and its Applications*, 369: 439–462, 2006. doi: 10.1016/j.physa.2005.11.053.

Tim P. Morris, Ian R. White, and Michael J. Crowther. Using simulation studies to evaluate statistical methods. *Statistics in Medicine*, 38(11):2074–2102, 2019. doi: 10.1002/sim.8086.

- Belinda Phipson and Gordon K. Smyth. Permutation P-values should never be zero: Calculating exact P-values when permutations are randomly drawn. *Statistical Applications in Genetics and Molecular Biology*, 9(1), 2010. doi: 10.2202/1544-6115.1585.
- Steven M. Pincus. Approximate entropy as a measure of system complexity. *Proceedings of the National Academy of Sciences*, 88:2297–2301, 1991. doi: 10.1073/pnas.88.6.2297.
- Alfred Renyi. On measures of entropy and information. In *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, volume 1, pages 547–561, 1961.
- I. A. Rezek and S. J. Roberts. Stochastic complexity measures for physiological signal analysis. *IEEE Transactions on Biomedical Engineering*, 45:1186–1191, 1998. doi: 10.1109/10.709563.
- Joshua S. Richman and J. Randall Moorman. Physiological time-series analysis using approximate entropy and sample entropy. *American Journal of Physiology—Heart and Circulatory Physiology*, 278:H2039–H2049, 2000. doi: 10.1152/ajpheart.2000.278.6.H2039.
- O. A. Rosso, H. A. Larrondo, M. T. Martin, A. Plastino, and M. A. Fuentes. Distinguishing noise from chaos. *Physical Review Letters*, 99:154102, 2007. doi: 10.1103/PhysRevLett.99.154102.
- Claude E. Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27:379–423, 1948. doi: 10.1002/j.1538-7305.1948.tb01338.x.
- Constantino Tsallis. Possible generalization of boltzmann-gibbs statistics. *Journal of Statistical Physics*, 52:479–487, 1988. doi: 10.1007/BF01016429.
- Wenxin Xia and Fang Wang. High-dimensional sample entropy for uncovering rich complex structures in data. *Chaos, Solitons and Fractals*, 208:118076, 2026. doi: 10.1016/j.chaos.2026.118076.