BEGINNING OF TRANSMISSION

State: unpublished / pending pairing

Release metadata

- **Title:** Active Inference Multi-Track Exemplar
- **Version:** 0.2.0
- **DOI:** 10.5281/zenodo.20417021
- **GitHub:** docxology/template_active_inference
- **Zenodo:** https://zenodo.org/records/20417021
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- **SHA-512:** `887a6f4f526c90c85d51db57d8205e8c9358e1a6352a58620db1c16842628ca7c028c3a16b4b05eb831b6eec1de47f98b5dcf8a21573552f270c28764fda9f26`

Pairing: pending — unresolved: - GitHub release URL: `pending`

Metadata Citation Contact Integrity Code128
Zenodo GitHub Manifest T:Active Inference Multi-Track Exemp

Figure 1: Integrity QR strip

Transmission manifest

```
title=Active Inference Multi-Track Exemplar
version=0.2.0 doi=10.5281/zenodo.20417021
sha256=e8b54ee92a09e8f2… manifest={"t":"Active Inference Multi-T","v":"0.2.0","d":"10.5281/zenodo.20417021","s":"e8b54ee92a09e8f2"}
```

Structured manifest: `../data/transmission_manifest.json`

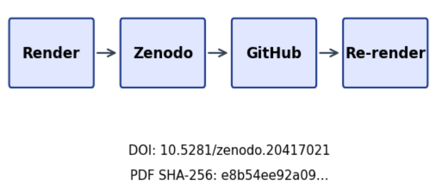

Figure 2: Publication pairing flow

Stego: off | overlays text | barcodes on | XMP on | manifest on → `./secure_run.sh`

Active Inference Multi-Track Exemplar

Sheaf-Composed Manuscript with pymdp Sophisticated Inference

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Contents

1 Sheaf Track Coverage

This page summarizes which **sheaf fragment tracks** are bound for each IMRAD row in `manuscript/sheaf/manifest.yaml`. The matrix is regenerated at compose time.
Totals: 42 present / 42 bound / 0 missing (gray).

Color	Meaning
Black	Track **present** (bound and fragment exists)
White	**Absent** (not bound for this row)
Gray	**Missing** (bound but fragment file absent)

1.1 Introduction

- **Introduction** *(group)*
- **Motivation and Active Inference scope**
- **Exemplar contributions** ## Methods
- **Methods** *(group)*
- **Analytical Bernoulli–Ising toy**
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- **Lean boundary witnesses**
- **Sheaf composition pipeline** ## Results
- **Results** *(group)*
- **Mutual information sweep**
- **Free energy decomposition**
- **T-maze sophisticated inference rollout**
- **Invariant gate summary** ## Discussion
- **Discussion** *(group)*
- **Limitations and extensions** ## Appendix
- **Full sheaf track coverage (proof)**

Coverage overview. Sheaf track coverage matrix: 16 IMRAD rows × 10 fragment columns. Black = present (P), white = absent (—), gray = missing (M). Counts: 42 present / 42 bound / 0 missing.

Appendix row `16_appendix_full_sheaf.md` binds 9 fragment track types as a composability proof (registry defines 10 types; optional `layers` is methods-only).

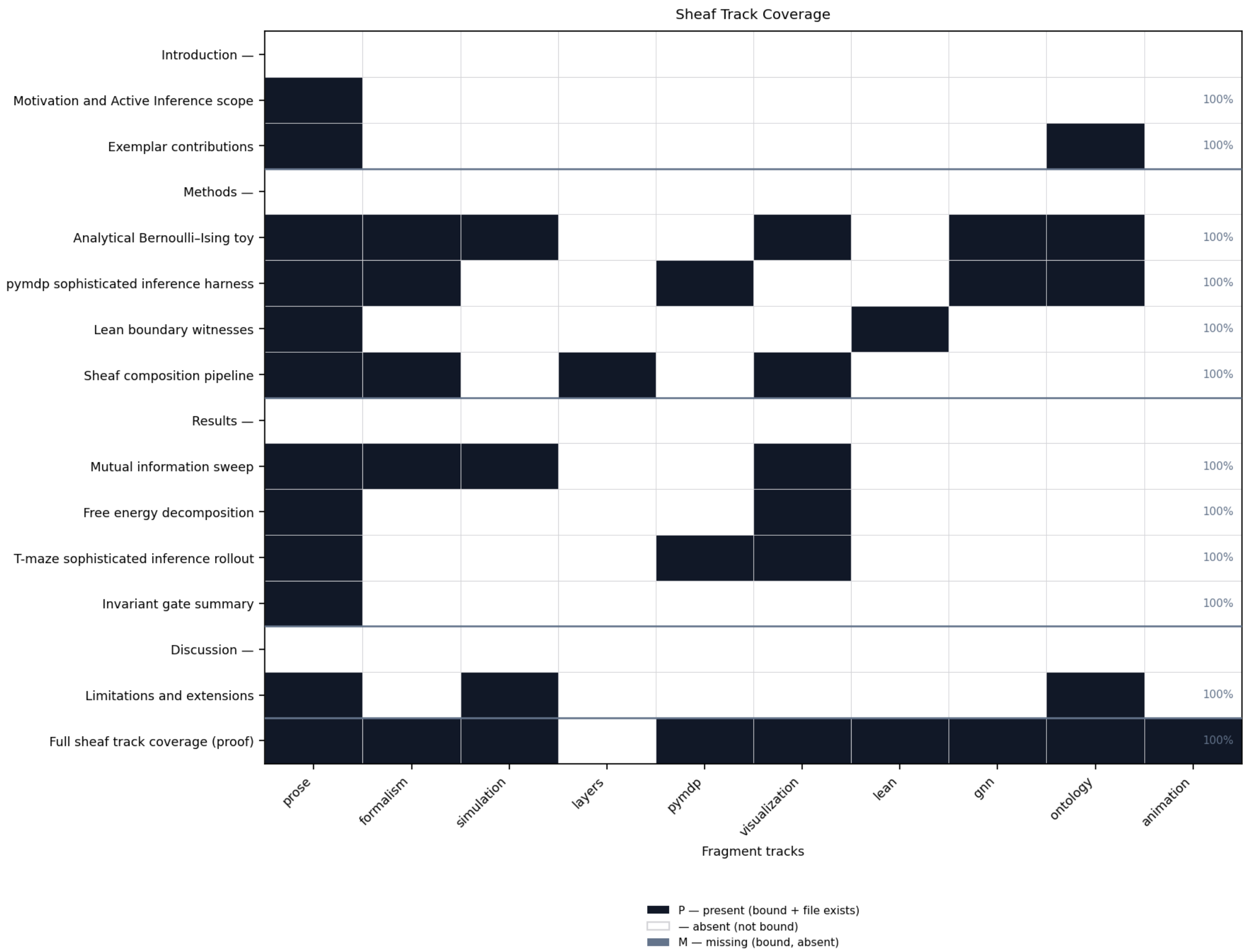

Figure 3: Heatmap matrix of IMRAD manuscript rows versus 10 sheaf fragment track columns. Black cells mean the track is bound and the fragment file exists; white cells mean the track is not bound; gray cells mean bound but missing. Rows are grouped by IMRAD block with indented subsection labels; column headers list track ids.

2 Abstract

This public exemplar binds **7 pipeline tracks** — Lean, analytical Python, pymdp sophisticated inference, GNN, ontology concordance, visualizations, and the sheaf manuscript composer — declared in `tracks.yaml`. Flat manuscript sections follow an **IMRAD outline** (Introduction, Methods, Results, Discussion, plus appendix) assembled from **10 sheaf fragment types** registered in `manuscript/sheaf/tracks.yaml`.
The first page (`00_00_sheaf_coverage.md`) shows a **16-row coverage matrix** (4 IMRAD group headers and 12 composed subsections, including a full-track appendix proof) regenerated from the live manifest at compose time.
The T-maze demo aligns with pymdp sophisticated_inference examples.
Measured invariant checks: 12 / 12 passed. SI planning horizon: 2 steps.

3 Motivation and Active Inference scope

This public exemplar demonstrates a **sheaf-composed** Active Inference manuscript: flat sections bind optional fragment tracks (prose, formalism, simulation, pymdp, visualization, Lean, GNN, ontology) under an IMRAD outline. The first page heatmap summarizes which tracks are bound per section.

The pymdp track follows the pymdp sophisticated_inference examples with a minimal T-maze and planning horizon `policy_len = 2`.

4 Exemplar contributions

This exemplar contributes:

1. **7 pipeline tracks** (`tracks.yaml`) — analytical Python, pymdp, GNN, ontology, Lean, visualizations, and sheaf composition — each with artifact gates.
2. **10 sheaf fragment types** registered in `manuscript/sheaf/tracks.yaml` (including optional `layers` and `animation`).
3. A **16-row IMRAD coverage matrix** on the first page, regenerated from the live manifest at compose time.
4. Registry-backed **invariant checks** (12 / 12 passed) and measured SI rollout metrics.

4.0.1 Ontology bindings

- `expected_free_energy` → **ExpectedFreeEnergy**
- `location` → **HiddenState**
- `observation` → **ObservationLikelihood**
- `policy` → **PolicyPosterior**

5 Analytical Bernoulli–Ising toy

We study a minimal **K=2 Bernoulli / Ising** coupling as the analytical companion to multi-track verification. Closed-form mutual information $I(\lambda)$ provides an oracle for empirical checks and GNN round-trips.
Measured sweep grid points: 21. Invariants passed: 12 / 12.
The entangled joint over binary policies satisfies

$$q_\lambda(\pi) \propto E(\pi)\, \exp(\lambda J(\pi)),$$

with symmetric Ising coupling J and deformation parameter λ. Mutual information is $I(\lambda) = \log 2 - H_b(\sigma(\lambda))$. The analytical track writes `output/data/parameter_sweep.csv` comparing closed-form and empirical mutual information across $\lambda \in [0, 4]$.

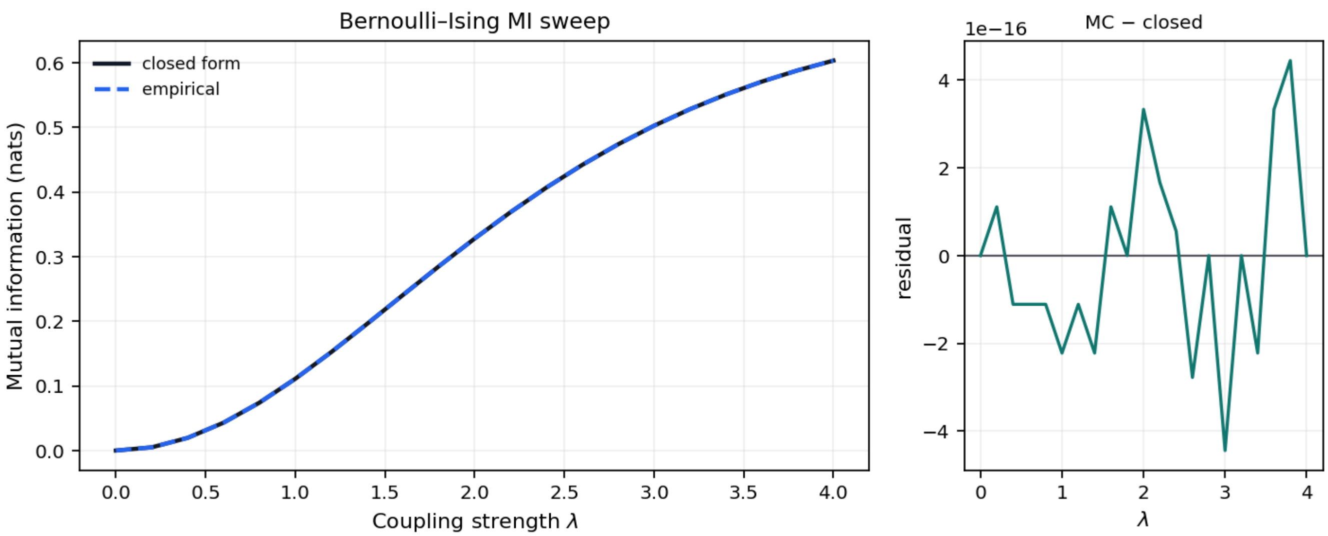

Figure 4: Two-panel plot of mutual information versus coupling strength lambda for the symmetric Bernoulli-Ising toy. Left panel shows closed-form curve (solid dark line) and Monte Carlo estimates (dashed blue line) with lambda on the x-axis and mutual information in nats on the y-axis. Right panel shows Monte Carlo minus closed-form residuals versus lambda with a zero reference line.

Figure 1 (methods). Closed-form and Monte Carlo mutual information $I(\lambda)$ for the symmetric Bernoulli-Ising toy across 21 grid points up to λ_max = 4; grid maximum 0.6031 nats on the measured sweep.
The Bernoulli toy is also declared in `gnn/bernoulli_toy.gnn.md` (GNN v1.1). Ontology annotations bind framework symbols to Active Inference Ontology terms.

5.0.1 Ontology bindings

- `J` → **CrossStreamCouplingPotential**
- `gamma` → **SophisticationWeight**
- `lam` → **EntanglementDeformationParameter**
- `pi1` → **Stream1PolicyVector**
- `pi2` → **Stream2PolicyVector**
- `q_joint` → **EntangledJointPosterior**

6 pymdp sophisticated inference harness

Sophisticated inference (planning horizon). This section documents a **real pymdp state-inference harness** on a minimal T-maze with planning horizon `policy_len = 2`. Default mode is `state_inference` (T-maze rollout via `simulate_si_tmaze.py`). The Agent constructs a multi-step policy set (`num_policies` logged in artifacts); per-step **belief entropy** is recorded in `output/logs/pymdp_runs.jsonl` and aggregated as `mean_belief_entropy`. Graph-world `infer_policies` is an opt-in extension stub — see `tracks.yaml` `extension_tracks.graph_world` and `scripts/simulate_si_graph_world.py`. Reference notebooks: pymdp sophisticated_inference.
Mean belief entropy across steps: 0.3251.
Given generative matrices A, B, C, D, pymdp computes state beliefs $q(s)$ via variational inference (`infer_states`). The Agent is configured with planning horizon $H = 2$, which defines the **policy depth** used when constructing candidate policies (logged as `num_policies` in `output/data/si_tmaze_summary.json`).
This exemplar records belief entropy per step; extending to full expected-free-energy policy selection (`infer_policies`) is documented as a follow-on track.
Artifact paths:

- `output/logs/pymdp_runs.jsonl` — append-only JSONL run log
- `output/data/si_tmaze_summary.json` — step count, actions, mean belief entropy
- `output/data/si_tmaze_trace.json` — rollout trace

Steps recorded: 2.
See `gnn/si_tmaze.gnn.md` for a GNN view of the T-maze hidden state, observation, and policy variables with ontology bindings.

6.0.1 Ontology bindings

- `belief_entropy` → **BeliefEntropy**
- `loc` → **HiddenState**
- `obs` → **ObservationLikelihood**
- `pi` → **PolicyPosterior**

7 Lean boundary witnesses

The Lean track provides minimal boundary witnesses checked by `lake build` under `lean/TemplateActiveInference/`. Fragments cite theorem names without duplicating proof scripts in prose.
See the Lean fragment for module references aligned with the analytical toy.
Lean module `TemplateActiveInference.SophisticatedInference` declares the planning horizon parameter and a witness that horizon > 1 distinguishes sophisticated from myopic inference.
Build via `lake build` under `lean/`.

8 Sheaf composition pipeline

Sheaf composition glues registered fragment tracks into flat manuscript sections using `manuscript/sheaf/manifest.yaml` and `manuscript/sheaf/tracks.yaml`.
The layers overview figure (registry caption in `figures.yaml section_figures.methods_sheaf`) summarizes the 10 fragment layer types and their IMRAD bindings in one panel (registry stack plus binding heatmap). The tables below list every track definition and section×track binding from the live manifest at compose time.

```
uv run python scripts/compose_manuscript.py
uv run python scripts/compose_manuscript.py --validate-only --strict
```

Each compose run emits `output/data/sheaf_coverage_matrix.json` and regenerates coverage artifacts. Partial compose (`--section`) is draft-only; the matrix always reflects the full manifest.
Coverage semantics: **black / P** = present (bound and file exists); **white / —** = absent (not bound); **gray / M** = missing (bound but file absent). Discussion limitations reference the same matrix counts (`42` / `42` / `0`).
Each fragment track $t \in \mathcal{T}$ maps to renderer $R(t)$ and compose-order index $\pi(t)$ from `manuscript/sheaf/tracks.yaml`. For manifest row s, cell $B(s,t) \in \{\mathrm{P}, \text{—}, \mathrm{M}\}$ follows the coverage matrix regenerated at compose time (42 present / 42 bound / 0 missing).
Registry size: $|\mathcal{T}| = 10$ types across 16 IMRAD manifest rows. Formal track definitions and section×track bindings appear in the generated tables below.
Figure 6 (methods). Sheaf layers overview: registry stack (compose order, renderer ids) and IMRAD binding heatmap for 10 fragment types across 16 manifest rows (42 present / 42 bound / 0 missing).

8.1 Sheaf fragment track registry

Compose order and renderer bindings from `manuscript/sheaf/tracks.yaml`.

Order	Track id	Label	Renderer	Optional
10	`prose`	Narrative prose	`markdown`	No
20	`formalism`	Mathematical formalism	`markdown`	No
30	`simulation`	Analytical simulation notes	`markdown`	No
35	`layers`	Sheaf layers tables	`layers_report`	Yes
40	`pymdp`	pymdp harness artifacts	`markdown`	No
50	`visualization`	Figure references	`section_figures`	No
60	`lean`	Lean boundary fragment	`markdown`	No
70	`gnn`	GNN notation fragment	`markdown`	No
80	`ontology`	Active Inference Ontology bindings	`ontology_yaml`	No
90	`animation`	Animation fragment	`markdown`	Yes

Track count: 10 registered fragment types.

8.2 IMRAD binding matrix

Section rows versus fragment track columns. **P** = present (bound and file exists); — = absent (not bound); **M** = missing (bound, file absent).

Section	prose	formalism	simulation	layers	pymdp	visualization	lean	gnn	ontology	animation
Introduction (group)	—	—	—	—	—	—	—	—	—	—

Section	prose	formalism	simulation	layers	pymdp	visualization	lean	gnn	ontology	animation
Motivation and Active Inference scope	P	—	—	—	—	—	—	—	—	—
Exemplar contributions	P	—	—	—	—	—	—	—	P	—
Methods (group)	—	—	—	—	—	—	—	—	—	—
Analytical Bernoulli–Ising toy	P	P	P	—	—	P	—	P	P	—
pymdp sophisticated inference harness	P	P	—	—	P	—	—	P	P	—
Lean boundary witnesses	P	—	—	—	—	—	P	—	—	—
Sheaf composition pipeline	P	P	—	P	—	P	—	—	—	—
Results (group)	—	—	—	—	—	—	—	—	—	—
Mutual information sweep	P	P	P	—	—	P	—	—	—	—
Free energy decomposition	P	—	—	—	—	P	—	—	—	—
T-maze sophisticated inference rollout	P	—	—	—	P	P	—	—	—	—
Invariant gate summary	P	—	—	—	—	—	—	—	—	—
Discussion (group)	—	—	—	—	—	—	—	—	—	—

Section	prose	formalism	simulation	layers	pymdp	visualization	lean	gnn	ontology	animation
Limitations and extensions	P	—	P	—	—	—	—	—	P	—
Full sheaf track coverage (proof)	P	P	P	—	P	P	P	P	P	P

Totals: 42 present / 42 bound / 0 missing.

Symbol	Coverage color	Meaning
P	Black	Track **present** (bound and fragment exists)
—	White	**Absent** (not bound for this section)
M	Gray	**Missing** (bound but fragment file absent)

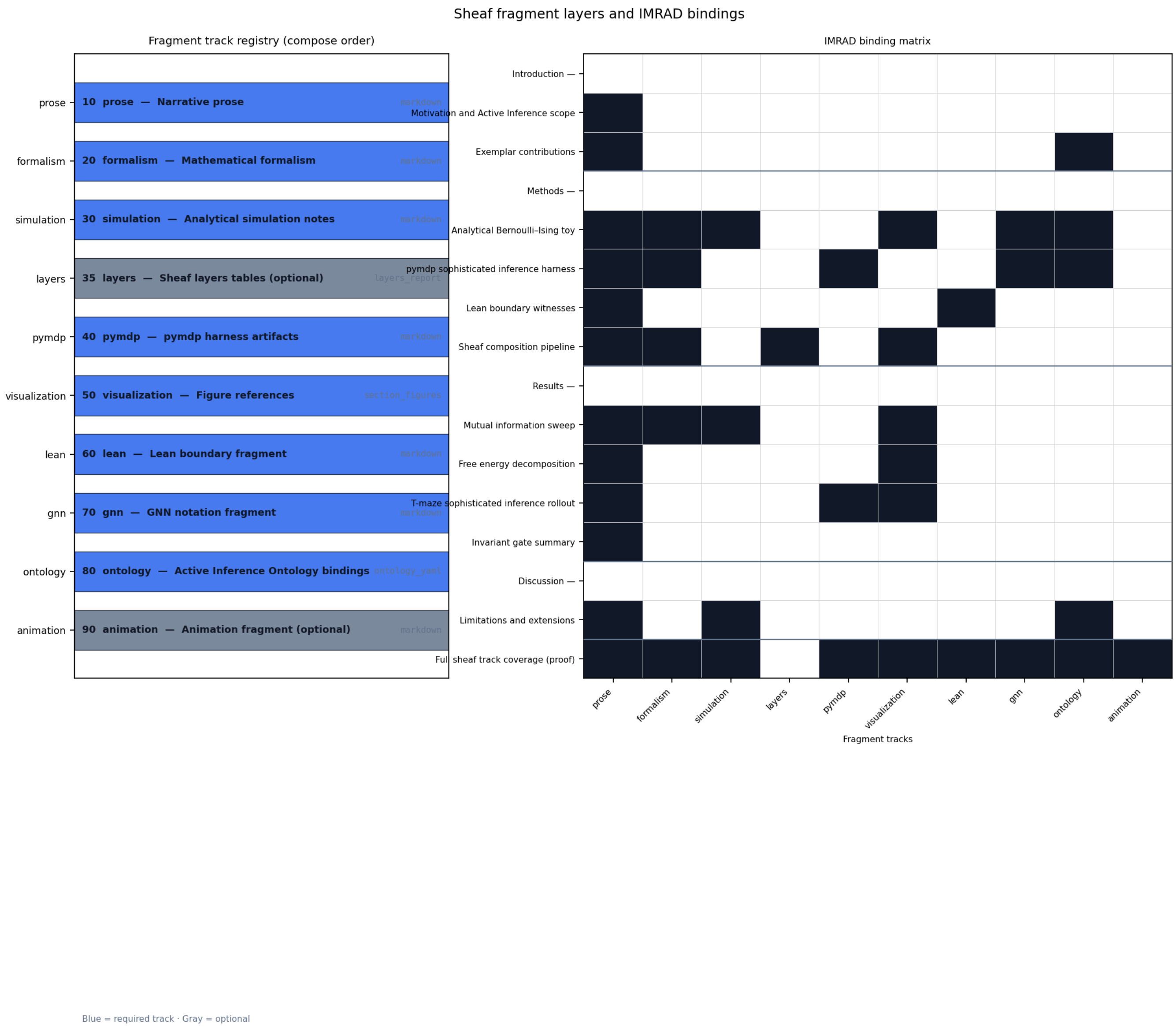

Figure 5: Two-panel overview of sheaf fragment layers. Left panel shows 10 composable track types in registry compose order with labels and renderer ids. Right panel shows the IMRAD section binding heatmap with black present, white absent, and gray missing cells across 16 manifest rows and 10 tracks.

9 Mutual information sweep

We sweep coupling strength λ on a grid of 21 points up to $\lambda_{\max} = 4$. Closed-form mutual information $I(\lambda)$ is compared to Monte Carlo estimates from the analytical module.
Measured invariant checks: 12 / 12 passed on the clean tree.
The entangled joint over binary policies satisfies

$$q_\lambda(\pi) \propto E(\pi)\, \exp(\lambda J(\pi)),$$

with symmetric Ising coupling J and deformation parameter λ. Mutual information is $I(\lambda) = \log 2 - H_b(\sigma(\lambda))$. Empirical MI estimates use fixed seed 0 and share the same λ grid as the closed-form sweep in `output/data/parameter_sweep.csv`.

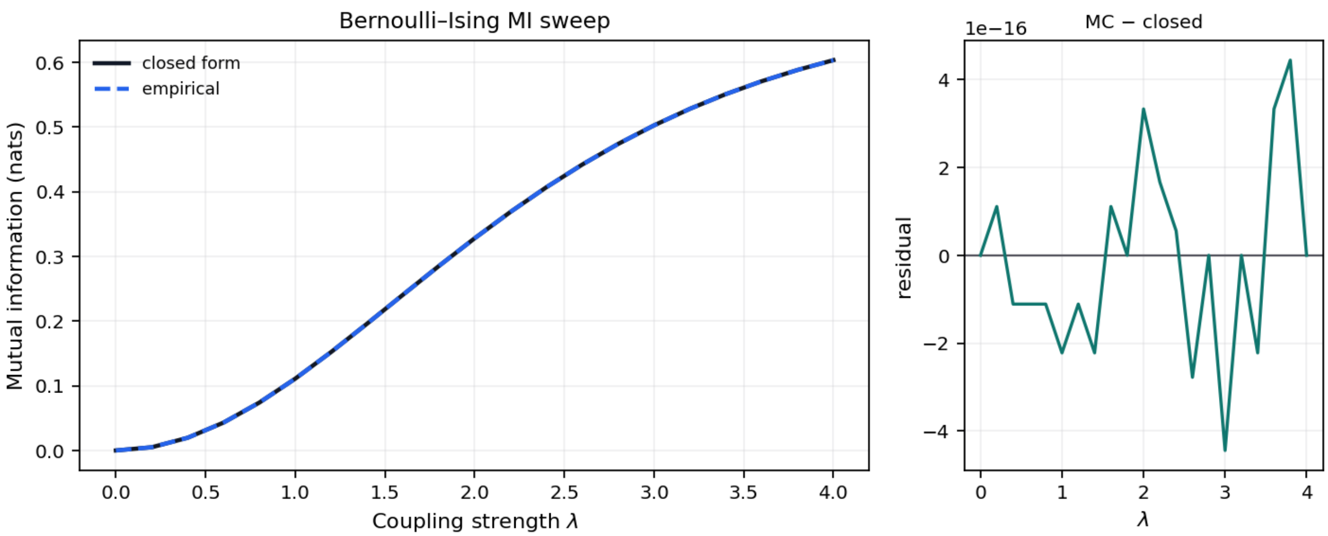

Figure 6: Two-panel plot of mutual information versus coupling strength lambda for the symmetric Bernoulli-Ising toy. Left panel shows closed-form curve (solid dark line) and Monte Carlo estimates (dashed blue line) with lambda on the x-axis and mutual information in nats on the y-axis. Right panel shows Monte Carlo minus closed-form residuals versus lambda with a zero reference line.

Figure 2 (results). Closed-form and Monte Carlo mutual information I(λ) for the symmetric Bernoulli-Ising toy across 21 grid points up to λ_max = 4; grid maximum 0.6031 nats on the measured sweep.

10 Free energy decomposition

Free energy against the entangled prior is evaluated along the same λ grid used for the MI sweep. The curve highlights how deformation strength trades off prior alignment and coupling energy in the analytical toy.
Saturation MI (grid maximum on the measured λ sweep): 0.6031 nats.

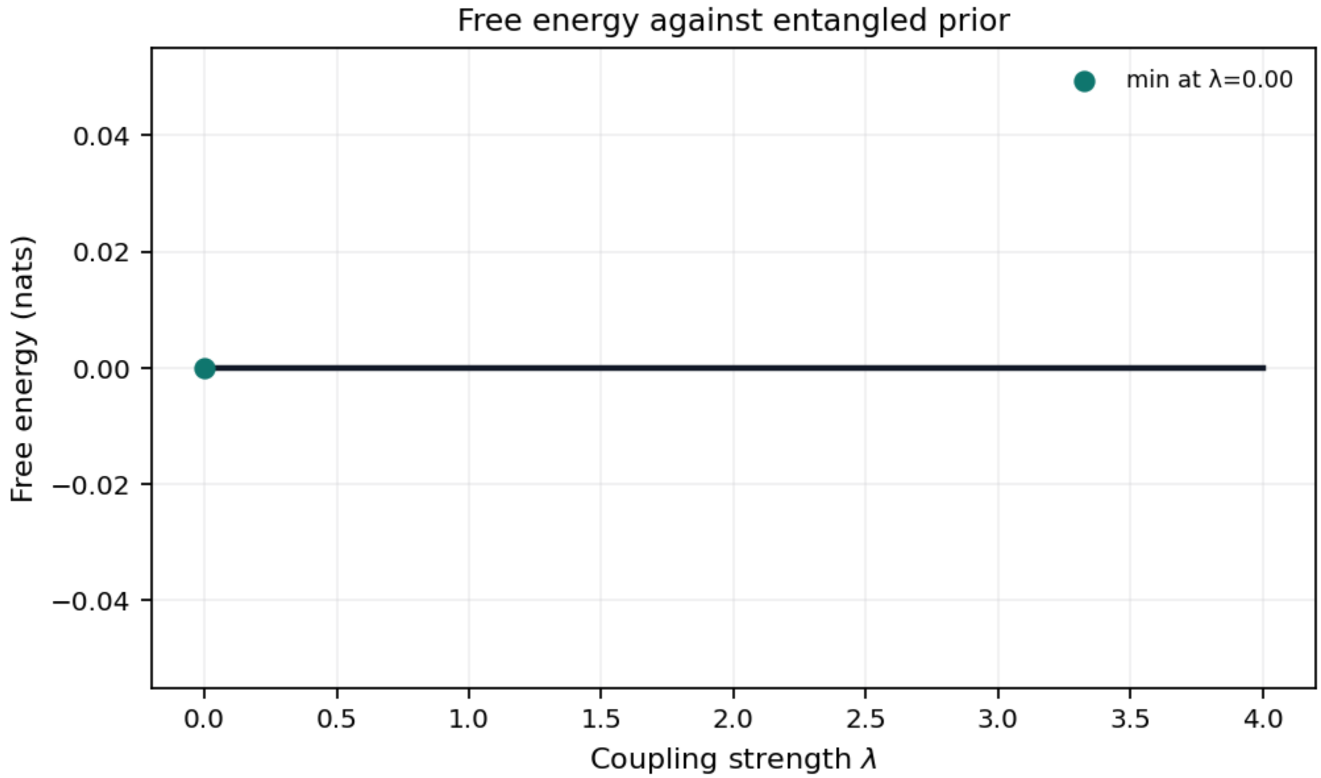

Figure 7: Line plot of free energy in nats versus coupling strength lambda for the entangled posterior relative to a mean-field prior. A single dark curve traces free energy across the sweep; the minimum is marked with a teal scatter point and labeled with the argmin lambda value.

Figure 3 (results). Free energy of the entangled posterior relative to the mean-field prior across the hyperparameter sweep (grid points 21).

11 T-maze sophisticated inference rollout

The pymdp harness rolls out a T-maze sophisticated-inference agent with planning horizon 2. Summary metrics land in `output/data/si_tmaze_summary.json`.
Steps recorded: 2. Mean belief entropy: 0.3251.
Rollout trace: `output/data/si_tmaze_trace.json`. JSONL run log: `output/logs/pymdp_runs.jsonl`.

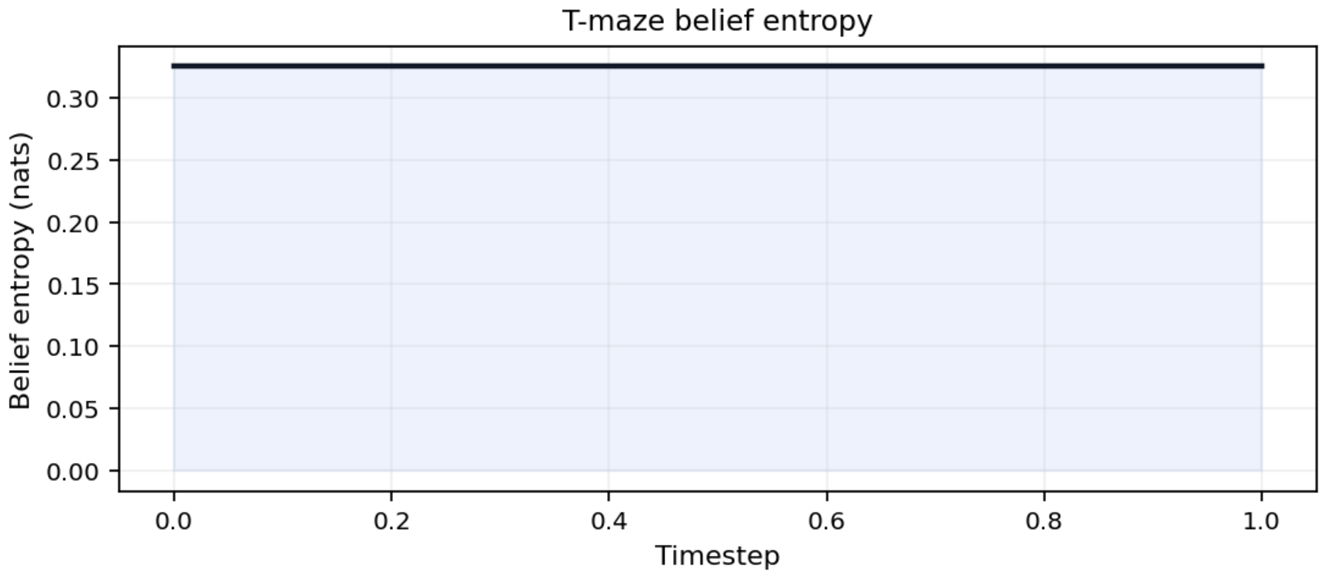

Figure 8: Line plot of belief entropy in nats versus timestep for the pymdp T-maze rollout. Entropy ranges from 0.3251 to 0.3251 nats across 2 steps in state_inference mode.

Figure 3a (results). Belief entropy over time for the T-maze rollout (mean 0.3251 nats).
Figure 3b (results). Observation and action traces for the T-maze rollout (action diversity 2).
Figure 3c (results). Discrete action index over time for the pymdp T-maze rollout (policy length 2).

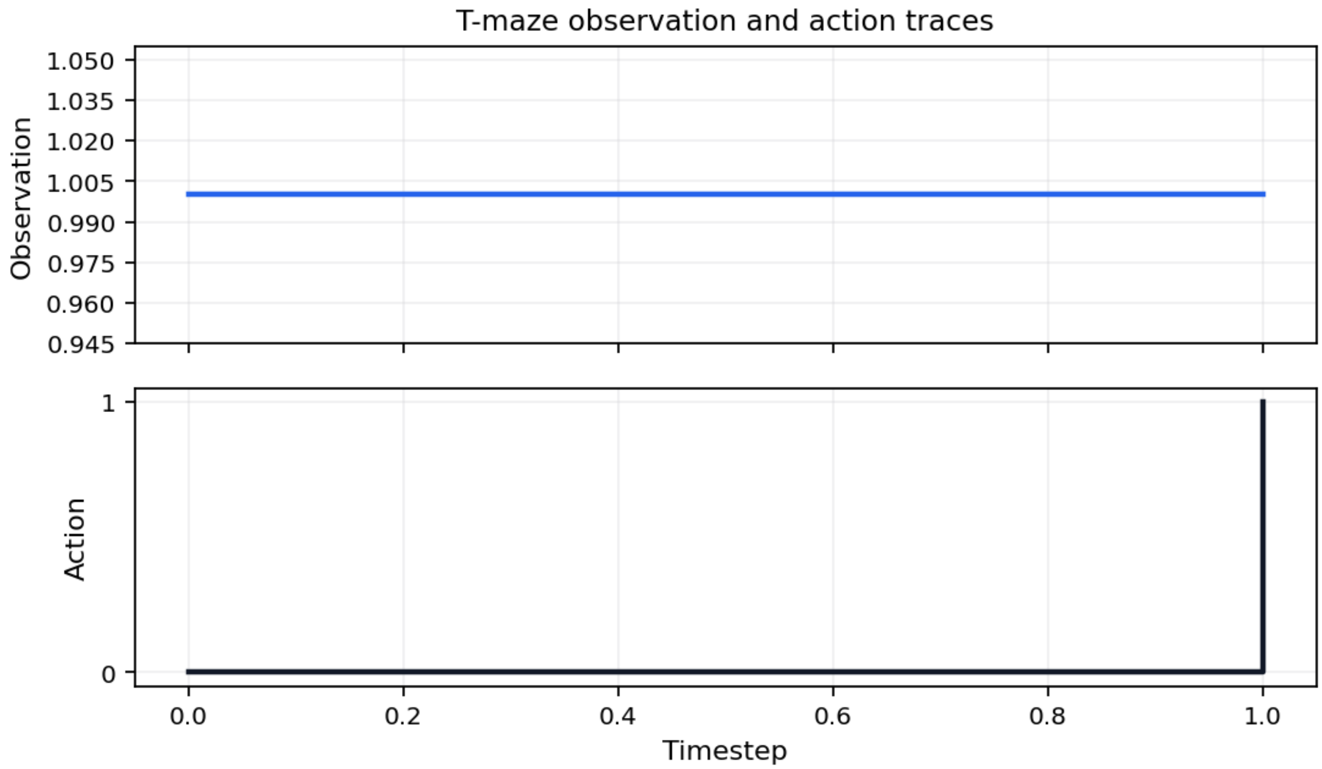

Figure 9: Dual-panel plot of observation index and action index versus timestep for the pymdp T-maze rollout. The upper panel shows discrete observations; the lower panel shows actions. Goal reached flag is 1.

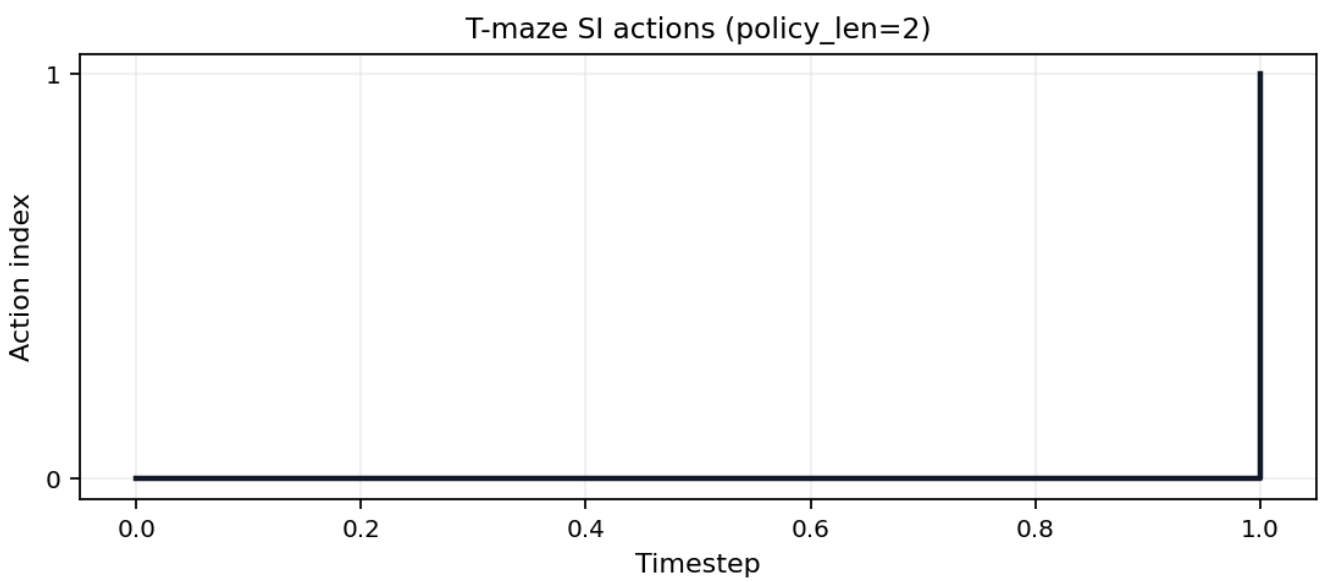

Figure 10: Step plot of discrete action index versus timestep for the pymdp sophisticated-inference T-maze rollout. Actions change at each timestep with light fill under the step trace; policy depth is 2 steps.

12 Invariant gate summary

The analytical invariant registry (`src/invariants.py`) runs before PDF rendering. On a clean checkout **12 / 12** analytical checks pass in the merged report at `output/reports/invariants.json`, which also records simulation invariants when the pymdp harness ran.

Per-domain SI checks live in `output/reports/si_invariants.json` before merge. Failures block publication artifacts.

13 Limitations and extensions

Limitations. The Bernoulli–Ising toy and T-maze harness are pedagogical — they validate pipeline wiring, not empirical claims about biological agents. Default pymdp mode is `state_inference` with planning horizon 2; full policy inference remains an opt-in extension.
Sheaf audit. The coverage heatmap and appendix 9-track proof make binding state auditable: 42 present / 42 bound / 0 missing cells across 16 manifest rows. Strict compose validation fails on gray cells so downstream PDF rendering inherits an honest matrix.
Extensions. Opt-in pipeline extensions in `tracks.yaml` `extension_tracks` (belief GIF via `render_animation.py`, graph-world SI stub) add artifacts outside the default analysis DAG. The default IMRAD manifest already binds an `animation` **sheaf fragment** on the appendix row; extension scripts are for richer media, not extra manifest rows. Promote a private project by copying the sheaf manifest pattern under `manuscript/sections/imrad/`.
Measured pymdp rollout (`state_inference`, config hash `ea4b126a9c22f22f`): mean belief entropy 0.3251 nats over 2 steps; goal reached flag 1; action diversity 2.
Analytical sweep residual RMSE 0 nats (max residual 0). Coverage audit: 42 present / 42 bound / 0 missing cells on the IMRAD matrix.

13.0.1 Ontology bindings

- `coverage_semantics` → **Coverage matrix semantics**
- `pedagogical_scope` → **Pedagogical scope**
- `state_inference_mode` → **State inference harness**

14 Full sheaf track coverage (proof)

This section is the **composability proof**: all 9 appendix-bound fragment tracks are rendered in one flat manuscript section. The registry defines 10 composable types; optional `layers` is methods-only and excluded from this row. The `animation` fragment is bound here (optional type in the registry) alongside prose, formalism, simulation, pymdp, visualization, lean, gnn, and ontology tracks.
For each track $t \in \mathcal{T}_{\text{Full}}$, the manifest binds a fragment path $f(t)$ and the composer emits <!-- sheaf-track:t --> before the rendered body.

$$|\mathcal{T}_{\text{Full}}| = 9$$

The fragment registry defines 10 composable track types; optional `layers` is bound on the methods sheaf section only. Optional `animation` is bound in this appendix proof; the opt-in GIF extension in `tracks.yaml extension_tracks` is separate from this fragment slot.
Analytical artifacts: `output/data/parameter_sweep.csv` and `output/reports/invariants.json` from the Bernoulli/Ising sweep track.
pymdp harness summary: `output/data/si_tmaze_summary.json` (mean belief entropy, action trace). Full log: `output/logs/pymdp_runs.jsonl`.

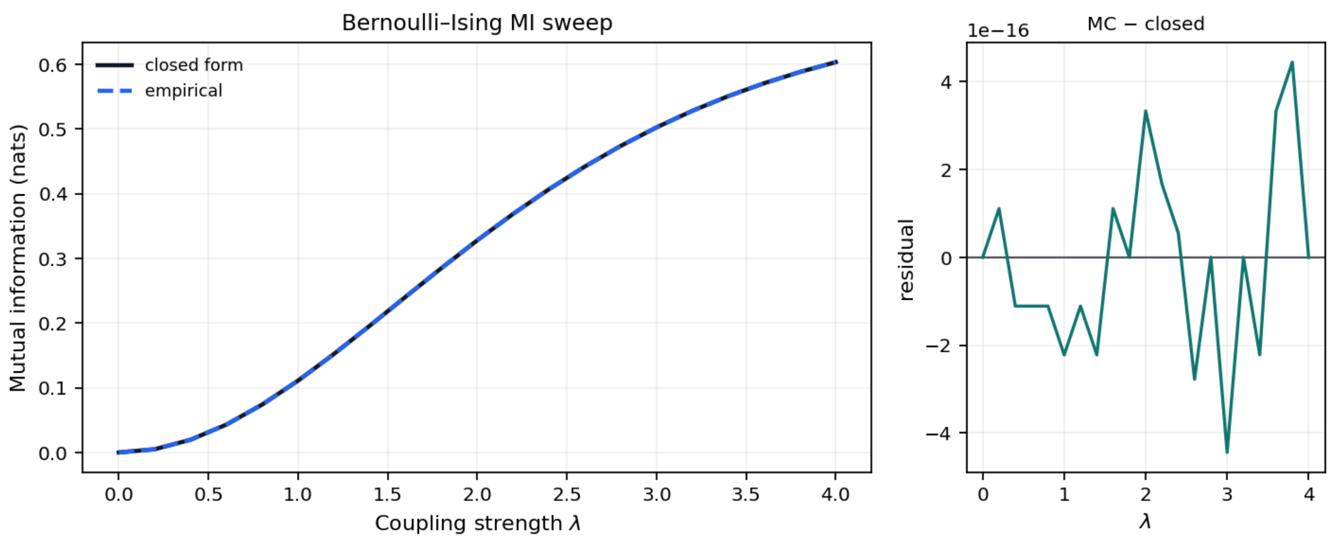

Figure 11: Two-panel plot of mutual information versus coupling strength lambda for the symmetric Bernoulli-Ising toy. Left panel shows closed-form curve (solid dark line) and Monte Carlo estimates (dashed blue line) with lambda on the x-axis and mutual information in nats on the y-axis. Right panel shows Monte Carlo minus closed-form residuals versus lambda with a zero reference line.

Figure A1 (appendix). Closed-form and Monte Carlo mutual information $I(\lambda)$ for the symmetric Bernoulli-Ising toy across 21 grid points up to λ_max = 4; grid maximum 0.6031 nats on the measured sweep.
Figure A2 (appendix). Discrete action index over time for the pymdp T-maze rollout (policy length 2).
Figure 4. Sheaf track coverage matrix: 16 IMRAD rows × 10 fragment columns. Black = present (P), white = absent (—), gray = missing (M). Counts: 42 present / 42 bound / 0 missing.
Lean modules under `lean/TemplateActiveInference/` declare horizon and coupling witnesses. Build with `lake build` in `lean/`; some lemmas remain `sorry` by design in this exemplar.
GNN declarations: `gnn/bernoulli_toy.gnn.md` and `gnn/si_tmaze.gnn.md`. Ontology concordance is validated against Active Inference Ontology bindings.

14.0.1 Ontology bindings

- `belief_entropy` → **BeliefEntropy**
- `expected_free_energy` → **ExpectedFreeEnergy**
- `location` → **HiddenState**
- `observation` → **ObservationLikelihood**

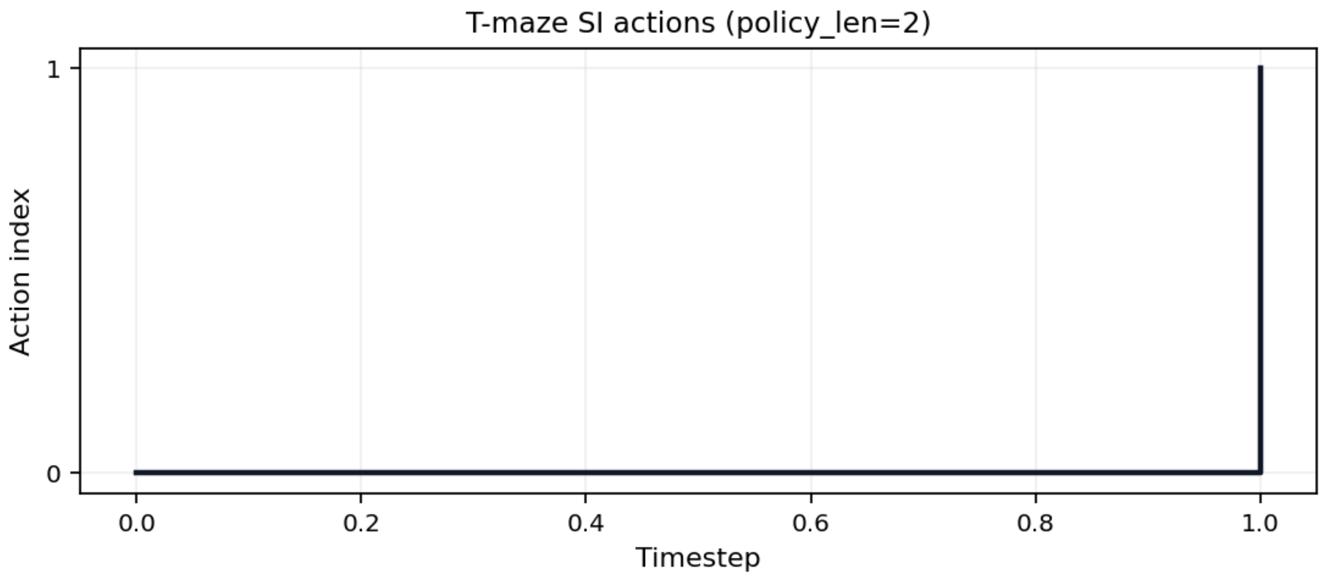

Figure 12: Step plot of discrete action index versus timestep for the pymdp sophisticated-inference T-maze rollout. Actions change at each timestep with light fill under the step trace; policy depth is 2 steps.

- `policy` → **PolicyPosterior**
- `sheaf_track` → **SheafFragment**

Animation **sheaf fragment** (bound in this appendix row): documents the optional registry track and points at static SI figure output.

Static frame artifact: `../figures/si_tmaze_actions.png` (from the default SI figure pipeline).

Opt-in **extension** GIF: run `scripts/render_animation.py` to write `../figures/si_belief_trajectory.gif` (placeholder loop from the belief-entropy curve until a true frame sequence lands under `../figures/animation/`).

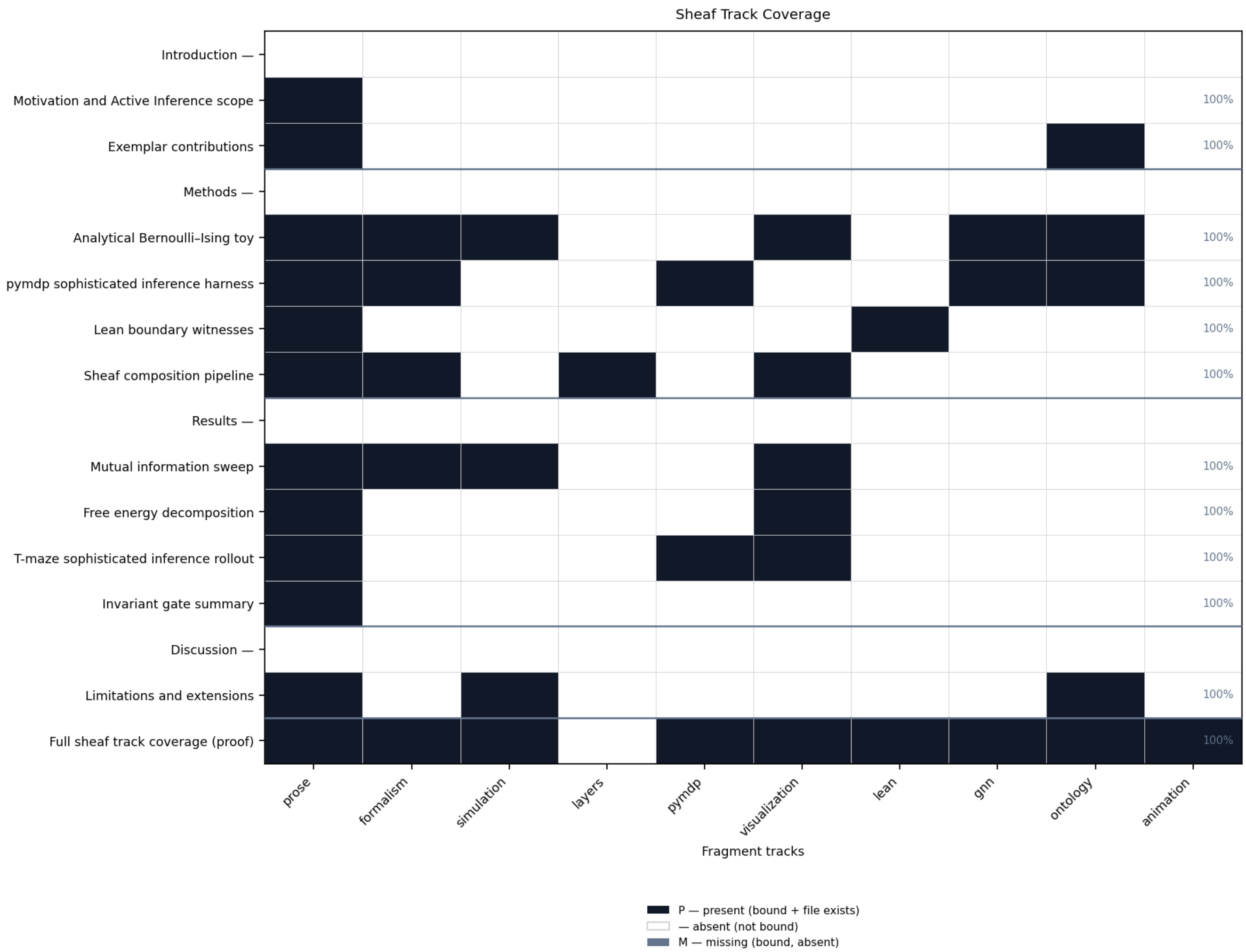

Figure 13: Heatmap matrix of IMRAD manuscript rows versus 10 sheaf fragment track columns. Black cells mean the track is bound and the fragment file exists; white cells mean the track is not bound; gray cells mean bound but missing. Rows are grouped by IMRAD block with indented subsection labels; column headers list track ids.

15 Conclusion

This exemplar shows how pipeline tracks, sheaf fragment registries, and IMRAD manuscript sections fit together: analytical oracles and pymdp rollouts produce measured artifacts; composition binds 12 flat sections from 10 fragment tracks; coverage JSON and the first-page heatmap report 42 present / 42 bound / 0 missing cells.
Strict validation (`compose_manuscript.py --validate-only --strict`) fails on gray matrix cells, keeping the appendix full-track proof and Results sections trustworthy for downstream PDF rendering. The T-maze harness runs in `state_inference` mode with config hash `ea4b126a9c22f22f`; sweep RMSE 0 nats bounds analytical–empirical agreement on the toy.

16 References

See `manuscript/references.bib` for bibliography entries cited in the composed sections.

References

END OF TRANSMISSION

Release: v0.2.0 · DOI 10.5281/zenodo.20417021 · SHA-256 e8b54ee92a09… · pairing pending

Metadata Citation Contact Integrity Code128

Zenodo GitHub Manifest T:Active Inference Multi-Track Exemp

Figure 14: Integrity QR strip

Prior: *No prior releases.*